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CURRICULUM REVISION PROGRAM

MATHEMATICS

Tentative Course of Study
for Junior High School
Grades 7 - 9

Supplementing
A Handbook and Guide for Teachers
With Tentative Goals for
Kindergarten Through Twelfth Grade

Public Schools of the District of Columbia
Washington, D. C.
1944

CURRICULUM REVISION PROGRAM
Carroll R. Reed
First Assistant Superintendent in Charge of Curriculum

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EDUCATION

TO OFFICERS AND TEACHERS OF THE PUBLIC SCHOOLS
OF THE DISTRICT OF COLUMBIA:

This course of study in junior-high-school mathematics is a supplement to the HANDBOOK AND GUIDE FOR TEACHERS WITH TENTATIVE GOALS FOR KINDERGARTEN THROUGH TWELFTH GRADE which was published in 1943. The HANDBOOK gives a unified picture of the philosophy and goals of the whole mathematics program. This course supplements the HANDBOOK by giving detailed content, references, and teaching suggestions for each course in the junior high school.

This junior-high-school course represents a cooperative achievement to which each junior-high-school teacher has had an opportunity to contribute. The Committee has worked earnestly and effectively to organize these contributions and present them in this form.

Much appreciation is due Miss Veryl Schult for her excellent professional leadership in this project, and Mrs. Ethel Grubbs for her fine cooperation and professional contributions. Through their untiring efforts and the fine cooperation of the chairman and members of the committee, this body of instructional material has been made available.

It is hoped that during the next school-year, when this course is being used experimentally, all teachers concerned, as a result of their experience with this course of study, will submit recommendations for the final revision.

Sincerely yours,

CARROLL R. REED

First Assistant Superintendent
in charge of Curriculum

COMMITTEE

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FOREWORD

TO TEACHERS OF JUNIOR HIGH SCHOOL MATHEMATICS:

The PURPOSES OF TEACHING MATHEMATICS on all levels are stated in the HANDBOOK. The materials and goals set up in this junior-high-school course of study are means of accomplishing these fundamental purposes.

The mathematics in the junior high school is a continuation and extension of the mathematics taught in the elementary school. Although a general level of achievement can be expected from entering junior-high-school pupils, all will not have reached the same level; thus the need for careful diagnosis and follow-up work. Teachers should familiarize themselves with the work of the lower and higher grades in order to provide for the best possible articulation. Help pupils grow in an understanding and appreciation of mathematics by continually pointing out the general principles which underlie and unify all branches of mathematics. The importance of generalizing can not be overemphasized.

The goals as stated for the junior-high-school courses include broad units of work for all pupils, but the more able the class, or the individuals, the more deeply they can go into the subject. The problem of individual differences among these pupils is cared for in various ways. In the first place, most junior high schools attempt to group all pupils homogeneously in their subject-matter classes. Within classes, teachers should: stress as a minimum in the allotted time those phases of the unit of work on which the pupils' success in the next higher grade depends; enrich the work of the unit for the brighter pupils. Many enrichment materials are mentioned in the column "Materials and References." All starred topics are optional and are another means of enriching the course for the more able. A standardized arithmetic test in the eighth grade for purpose of guidance into the appropriate ninth-grade mathematics course is recommended. Elementary Algebra is offered for those whose arithmetic achievement is satisfactory and who are planning to enter the academic course in the senior high school; General Mathematics is for slow pupils who are deficient in arithmetic achievement and who need another year of mathematics work before taking Algebra or any senior-high-school course. Business Arithmetic is for ninth graders who are in the commercial course.

The materials, references, and teaching suggestions are merely suggestive, and are by no means meant to be all-inclusive. Their use is left to the judgment of the teacher, but they represent ideas which some teachers have found helpful.

Teachers are urged to make use of historical phases of mathematics wherever possible. Very often, the historical approach to a new topic will not only increase the pupils' interest but also will help to develop more thorough understanding.

A working vocabulary of the necessary mathematical terms should grow out of each unit of work. For that reason, a vocabulary is not listed for each grade.

A very important phase of the work of each teacher is evaluation or appraisal of pupil progress. Refer to the article on Evaluation in the HANDBOOK.

Make proplem solving an important phase of each semester's work, using practical problems of current interest where possible. Some habits which are desirable in all problem solving are: reading the problem carefully to select what is given and what is required, drawing a diagram where possible, estimating a reasonable result, checking for accuracy.

Many teachers are not content with four walls and a blackboard, and are giving more attention to the furnishing and equipment of mathematics classrooms. When classrooms are enlivened by mathematical visual aids, instruction is aided and interest is stimulated. See the article in the Appendix "A Mathematical Atmosphere in the Classroom".

Finally, mathematics can be fun. Help pupils enjoy it.

7A

COURSE OF STUDY

Goals	Materials and References
I. INFORMAL GEOMETRY (3 weeks)	
A. An appreciation of the geometry of everyday life through a familiarity with, and the ability to sketch such basic forms as: 1. Rectangle 2. Square 3. Parallelogram 4. Triangle 5. Circle 6. Cube 7. Rectangular solid 8. Cylinder 9. Cone 10. Sphere	EXCURSION IN MATHEMATICS, by E. R. Breslich. Orthovis Co., Chicago, 1938. "Origin of Mathematics". 16 mm. sound film. Visual Education Department, Public Schools, D. C. "Mysteries of Snow" 16 mm. silent movie. Visual Education Department, Public Schools, D. C. A SOURCE BOOK OF MATHEMATICAL APPLICATIONS. Seventeenth Yearbook of National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, New York, N. Y., 1943. pp. 170-179. JUNIOR MATHEMATICS FOR TODAY, by Wm. Betz. Ginn and Co., 1938. "The World of Geometry" MATHEMATICS AT YOUR SERVICE, by Larry and Addleton. Globe Book Co., 1941. "Teaching of Intuitive Geometry," William Betz. Eighth Yearbook of National Council. "Geometry for Everyone," by Kenneth Davis. The Mathematics Teacher, Feb., 1942. "How Intuitive Geometry Serves the Community," by A. Brown Miller. The Mathematics Teacher, Dec., 1941.

Teaching Suggestions

I. INFORMAL GEOMETRY

A. Geometry may be a new word to some pupils. It will be interesting to them to learn the origin of the word (geo-earth, metry-measure), so called because one of its earliest and most important applications was to the measurement of the earth's surface. Point out that the geometry of land measurement was developed by the Egyptians in the Nile valley where yearly floods wiped out farm boundaries. This work necessitated the use of some geometric figures with which the pupils are familiar.

Let pupils find illustrations of the geometric forms in the classroom and in their immediate surroundings. Also have them look for them on their way to and from school and at home.

List the geometric figures pupils can name. Supply those not mentioned.

Discuss the geometry found in nature: snow flakes, honey combs, fruits, spider webs, crystals, shells, and the like.

Discuss the shapes of boxes, bottles, cans, etc. as examples of geometric figures.

Use flash cards for quick recognition of geometric forms.

Give pupils practice in drawing these figures, both freehand and with the ruler.

Use pictures of churches, public buildings, house interiors, etc. and identify the geometric forms found in them.

Collect materials showing geometric forms in oil cloth, dress materials, linoleum, wall paper, tiles, buttons, quilt patterns, lace, Mexican and Indian designs.

Mention the geometric forms found in primitive shelter such as the wigwam, igloo, and tepee.

Goals	Materials and References
I. (Continued)	"Ancient Mathematics and Primitive Culture," by William A. Cordrey. *The Mathematics Teacher*, Feb., 1939 *Flash Cards* showing geometric forms *Models* of wood, paper, clay, soap, or wire A model for a parallelogram can be made of 4 strips of cardboard or wood fastened together so that the corners pivot. *Kaleidoscope*

Teaching Suggestions

I. (Continued)

Goals	Materials and References
I. (Continued)	
B. A knowledge of the use of the ruler, protractor, and squared paper as geometric tools, acquired through: 1. Drawing and identifying the different kinds of lines: a. Straight, curved, broken b. Horizontal, vertical, oblique c. Parallel, intersecting, perpendicular	B. Blackboard Tools: Compass, protractor, T-square, straight-edge, triangle, graph chart For pupils: Ruler, protractor, graph paper, scissors, colored chalk "Objective Materials in Junior High School Mathematics" by J. Mahachek, The Mathematics Teacher, Oct.,1939.
2. Measuring and drawing angles: a. Acute b. Right c. Obtuse d. Straight	A device formed by fastening together two strips of cardboard or wood is useful in teaching angles. A clock face with movable hands THE PLACE OF MATHEMATICS IN SECONDARY EDUCATION; Fifteenth Yearbook of National Council of Teachers of Mathematics. Pp. 227-228.

Teaching Suggestions

I. (Continued)

B.

1. Develop the concept that a line is indefinite in length and has only one dimension. A line segment is the part of a line between two points.

   Give practice in drawing and identifying line segments according to their position and relation to other lines.

   Familiarize pupils with notation used in labeling line segments.

   Find illustrations of various kinds of lines in the classroom, on the street, and in buildings.

2. Develop the concept of angle. Use scissors, hands of clock, etc.

   Teach how to label and read angles by three letters and also by a single letter.

   Classify angles according to size.

   Point out why there are 360° in a circle. Compare the use of minutes and seconds in measuring angles with their use in measuring time. Correlate with latitude and longitude.

   Give practice in:
      a. Estimating angles of 45° and 90°
      b. Measuring angles with protractor
      c. Drawing angles with the protractor

Teaching Suggestions

I. (Continued)

B.

3. Have pupils make geometric designs using lines and angles with which they have become familiar.

   Paper folding will also produce many interesting designs. See reference.

   In the study of informal geometry, the following words should become a part of the pupil's vocabulary, both oral and written:

   rectangle
   square
   parallelogram
   triangle
   horizontal
   perpendicular
   oblique
   circle
   cube
   prism
   acute
   right
   degree
   cylinder
   cone
   sphere
   angle
   vertex
   obtuse
   straight
   curved
   broken
   vertical
   parallel
   intersecting

Goals	Materials and References
II. ARITHMETIC (8 weeks)	ARITHMETIC IN GENERAL EDUCATION; Sixteenth Yearbook of National Council of Teachers of Mathematics. Chapter 8.
A. An appreciation of the development of our number system through:	
1. Wide experience with numbers in meaningful situations	
2. Use of historical materials.	NUMBER STORIES OF LONG AGO, by D. E. Smith; Ginn and Co. 1929. THE WONDERFUL WONDERS OF ONE-TWO-THREE, by D. E. Smith; McFarlane Ward McFarlane, 1937 NUMBERS AND NUMERALS, by Smith and Ginsburg; Bureau of Publications, Teachers College, Columbia Univ. 1937. STORY OF NUMBERS. American Council of Education. An abacus made of sand and pebbles or beads and wire.

Teaching Suggestions

II. ARITHMETIC

A. 1. Have pupils try to describe their experiences during a day without mentioning number.

Experiences with numbers in meaningful situations can be provided through problems involving time, money, distance, speed, age, sizes of clothes and shoes, quantities, weights, etc.

Give practice in reading and writing large numbers having social significance. Daily newspapers furnish current material for this.

Emphasize correct method of reading numbers. "And" should be used only for the decimal point.

2. Show pupils how mathematics has developed with civilization.

Discuss early devices for counting as: fingers, pebbles, notches on sticks, knots on strings, abacus, etc.

Contrast number symbols of the Egyptians, Babylonians, Romans, and Chinese with our numerals.

Show the advantage of our number system because of:

   a. The place value of digits.
   b. The use of the zero.

Show the advantage of our number system over Roman by multiplying two numbers such as:

$$\begin{array}{r} 487 \\ \times\ \underline{35} \end{array} \qquad\qquad \begin{array}{r} \text{CDLXXXVII} \\ \times\ \underline{\qquad\text{XXXV}} \end{array}$$

Goals	Materials and References
II. (Continued)	
[illegible]. Increased ability in performing the four fundamental operations with integers, common fractions, and decimal fractions, obtained through:	
1. Purposeful review based on the results of inventory tests	Standardized Tests that are available through the Research Department or in the textbooks in use
2. The use of more advanced material such as long division with three digit divisors.	ARITHMETIC FOR TEACHERS, Roantree and Taylor. Macmillan, 1938. MATHEMATICS IN AVIATION, by Ostoyees Macmillan, 1942. Chapter 10 (Averages) MATHEMATICS IN DAILY USE, Hart, Gregory, Schult, D. C. Heath Co., 1940, Chapters I and II.

Teaching Suggestions

II. (Continued)

B.

1. Give diagnostic tests to determine weaknesses in fundamentals.

   Analyze the common errors. Reteach for mastery.

   Give drills at regular intervals to maintain skill in fundamental processes.

   Give practice in finding averages.

   The ability to add or subtract fractions arranged horizontally will prove helpful in shop work.

2. Stress the use of the trial divisor and correct placement of digits in quotient.

Goals	Materials and References
II. (Continued)	
3. Familiarity with the relationship existing between common and decimal fractions and the ability to change from one to another	
4. Practice in making mental calculations, estimating results, and checking for accuracy	
5. Applications to problems, arising in the home, store, community and school (science, shops, homemaking classes)	"Facts and Figures in Making Problems," *The Mathematics Teacher*, Dec. 1943; p.373

Teaching Suggestions

II. (Continued)

3. Emphasize the fact that common and decimal fractions express the same concepts. Try to show that new ideas are developed in mathematics to make work easier.

   Fractions in most common use are halves, fourths, eighths, sixteenths, thirty-seconds, sixty-fourths, thirds, fifths, sixths, tenths, twelfths. Stress these.

   Have pupils make a three or four place conversion table changing these fractions to decimals. Such a table can be posted on the bulletin board and referred to as is done in shop work.

5. Use problems related to school, home, and community for practice in fundamental processes, such as:
   a. Cost of classroom equipment, books, supplies, etc.
   b. Increasing and decreasing recipes.
   c. Cost of operating a home with rent, heat, light, telephone, etc.
   d. Cost of equipment and clothing for an enlisted man.
   e. Cost of a balanced school lunch.
   f. Computing weekly wages.

Goals	Materials and References
II. (Continued)	
C. An awareness of the use of percentage in the business world and a thorough understanding of some of the basic concepts involved, such as: 1. The meaning of the % sign	MATHEMATICS IN ACTION, by Hart and Jahn, Bk. I., D. C. Heath Co., 1939. Chapter 6. EVERYDAY JUNIOR MATHEMATICS, Bk. I; by Betz. Chapter 7. FUNDAMENTAL MATHEMATICS, Bk. I, by Schorling and Clark, World Book Co., 1944, Unit V.

II. (Continued)

C. Discuss with pupils the uses of percentage in everyday life.

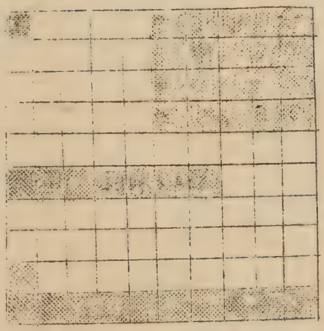
1. Have pupils bring in items from newspapers and magazines showing the use of the % sign.

   Introduce the meaning of the % sign as a symbol for expressing fractions with denominators 100.

   Impress upon pupils the meaning of % by working with a square containing 100 squares such as the one below:

$\frac{1}{100} = 1\%$

$\frac{20}{100} = 20\%$



$\frac{7}{100} = 7\%$

$\frac{11}{100} = 11\%$

2. Have pupils prove equivalents, such as:

   $50\% = \frac{1}{2}$ $\quad$ $(50\% = \frac{50}{100} = \frac{1}{2})$

   $25\% = \frac{1}{4}$ $\quad$ $(25\% = \frac{25}{100} = \frac{1}{4})$

Goals | Materials and References

II. (Continued)

2. The relationship between per cent, common fraction and decimal fraction, with the following fractional equivalents made automatic:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$,

$\frac{1}{5}$, $\frac{1}{10}$

D. The ability to find a per cent of a number by:

1. Using fractional equivalents for: 50%, 25%, $33\frac{1}{3}\%$

$66\frac{2}{3}\%$, 20%, 10%.

2. Using decimal equivalents for any other

E. The ability to use percentage in everyday problems involving the finding of a per cent of a number

Teaching Suggestions

II. (Continued)

Develop the ability to change a fraction to a per cent:

$$\frac{1}{2} = \frac{50}{100} = 50\% \qquad \frac{1}{2} = \begin{array}{r} .50 \\ 2\overline{)\,1.00} \end{array} = 50\%$$

$$\frac{1}{5} = \frac{20}{100} = 20\% \qquad \frac{1}{5} = \begin{array}{r} .20 \\ 5\overline{)\,1.00} \end{array} = 20\%$$

Drill in interchanging fractions, decimals, and per cents.

D.

1-2. Give much practice (including oral practice) in examples of the following types:

a. 75% of 96 = ?

$\frac{3}{4} \times 96 = 72$

Thus, 75% of 96 = 72

c. ? is 80% of 45

$\frac{4}{5} \times 45 = 36$

36 is 80% of 45

b. 14% of 758 = ?

$$\begin{array}{r} 758 \\ \underline{\times .14} \\ 3032 \\ \underline{758} \\ 106.12 \end{array}$$

E. Include problems made up by the pupils.

Test pupils' vocabulary of arithmetic orally and in writing in order to determine their familiarity with the vocabulary of arithmetic.

Goals	Materials and References
III. MEASUREMENT (6 weeks)	
A. An appreciation of the need for standard units of measurement as brought out in the history of the development of the common units of length, both English and Metric	MATHEMATICS IN LIFE, by Schorling and Clark. World Book Co., 1937. Pp.17-56 "A Study of Weights and Measures," by G. C. Yorke The Mathematics Teacher, March, 1944. "A Study of Weights and Measures," by J. T.Johnson, The Mathematics Teacher, May, 1944. THE STORY OF WEIGHTS AND MEASURES: American Council on Education. Pictures of the History of Measurement available in various places (How Long is a Rod? He Measures to Millionths Ford Motor Co.)
B. The ability to measure lengths accurately, acquired through the use of the ruler, and ability to estimate	"The Teaching of Direct Measurements in the Junior High School," by Betz, THIRD YEARBOOK OF THE NATIONAL COUNCIL. pp.149-194. "The Teaching of Approximate Computation," by West and Shuster, The Mathematics Teacher, Apr.,1943.

Teaching Suggestions

III. MEASUREMENT

A. Give brief presentation of the history of the development of the units of length. Show the need for standard units of length.

Include the interesting historical facts about the development of the metric system.

B. Give pupils practice in estimating lengths in the classroom and on the school grounds. Include lengths less than an inch. Use the span and pace to help in estimating. Check with actual measurement.

Show that all measurements are approximate. Point out that the degree of accuracy in measuring depends upon what is measured, the reason for measuring, and instruments used. For rules concerning the operations with approximate numbers, see the reference on "Teaching Approximate Computation".

Goals	Materials and References
III. MEASUREMENT (Continued)	Foot ruler, yard stick, meter stick, tape measure, 50-ft. tape, Metric Chart (available from the Gov't Printing Office and from D. C. School Supply List, No. 29-F-1); strip of gummed paper marked off in feet and inches pasted over blackboard molding; squared paper for finding perimeters and areas Graph blackboard chart.
C. An understanding of the meaning of: 1. Perimeter 2. Area	
D. A knowledge of the tables of linear and square measure	
E. The ability to find the perimeter and area of the following figures by using the formula: 1. Rectangle 2. Square 3. Triangle	
F. An understanding of the use of the perimeter and area developed through solving significant problems	

III. MEASUREMENT
(Continued)

Have pupils study the ruler and yard stick to learn the divisions on them.

Have pupils build and learn the table of linear measure.

Give practice in measuring objects which require using fractional parts of a unit.

C-D-E-F

Perimeter

Make actual measurements in developing the concept of perimeter.

Show the derivation of the word perimeter (peri, around; meter, measure)

Develop rules for finding the perimeter of rectangle, square, and triangle. Translate rules into formulas.

Suggestions for evaluating a formula:
1. Copy the formula
2. Write what is given
3. Substitute numbers for letters and perform computations.
4. Label the answers

Make applications to problems involving fencing gardens, framing pictures, measuring for lace edgings, picture moulding, base boards, etc.

Goals	Materials and References
III. MEASUREMENT (Continued)	

Teaching Suggestions

III. MEASUREMENT (Continued)

Area

Use squared paper to show that area means the number of square units on a surface.

Develop the table of square measure through drawings or models of a square inch, a square foot divided into square inches, and a square yard divided into square feet. Have pupils learn table of square measure.

Develop rules for finding areas of rectangles, squares, and triangles. Translate rules into formulas for area.

Emphasize the fact that perimeter is in linear units and area is in square units.

Suggestive data for problems:
- Linoleum and broadloom carpets are bought by the square yard.
- Sod and screening are bought by the square foot.
- Estimates for painting are made by the square foot.
- Building lots are sold by the square foot.

7B

COURSE OF STUDY

Goals	Materials and References
I. GRAPHIC REPRESENTATION (3 weeks)	
A. An appreciation of the value of graphs for picturing numerical data developed through reading and interpreting pictographs, bar, line, and circle graphs	MATHEMATICS IN LIFE by. Schorling and Clark. World Book Co., 1937. Unit V.
B. Ability to construct bar, line, and circle graphs	"Towards Better Graphs" by E. Eagle. The Mathematics Teacher, March, 1942. Blackboard graph chart. Squared paper. JUNIOR MATHEMATICS FOR TOADY, Bk. I, by Betz, Ginn & Co., 1938, Chapter IX. MATHEMATICS IN ACTION, Bk. I, by Hart & Jahn. D. C. Heath, 1939, Chapter VII. FIRST COURSE IN THE NEW MATHEMATICS, by Edgerton & Carpenter. 1939. Unit III.

Teaching Suggestions

I. GRAPHIC REPRESENTATION

A. Present graphs as a type of picture with a story to tell. Consider the widespread use of graphs. It is more important for pupils to read and interpret graphs than to be able to make them.

Have pupils bring in graphs from newspapers, magazines, and advertisements from industry for study and interpretation. Also use graphs found in history and geography textbooks.

It will be interesting for pupils to know that there are instruments that draw graphs, such as the cardiograph, and self-registering thermometer.

B. In the construction of graphs emphasize the following:
1. The graph should have a title which should be complete and clearly stated, so that misinterpretation would be impossible.
2. The source of the numerical data used should be indicated on the graph. In many cases it is wise to show the data also.
3. In picturing areas or volumes to represent quantities, care should be exercised to prevent giving false impressions. This applies especially to pictograph area and volume pictures.
4. On bar graphs and line graphs the zero line should be included or its omission clearly indicated.
5. The scales on a line graph should be so arranged and so numbered that an increase is represented by a rising of the line and decrease by a dropping of the line.
6. On multiple bar graphs the direction of the bars should be consistent with the data represented.
7. A type of graph should be selected that logically portrays the information.

Goals	Materials and References
I. GRAPHIC REPRESENTATION (Continued	

Teaching Suggestions

I. GRAPHIC REPRESENTATION
(Continued)

8. The use of appropriate colors and shadings should be taught and encouraged.
9. Artistic contrast and artistic composition should be emphasized. *

* From THE MATHEMATICS TEACHER, March, 1942. "Toward Better Graphs," by E. Eagle.

Select data meaningful to pupils, such as:

Attendance and tardiness
Temperature
Results of school competitive campaigns:
P.T.A. membership, school paper subscriptions, sale of tickets, etc.
Contributions to the Red Cross made by various sections.
Test results
Geographical facts
How I spend my day
How I spend my money

Strips of colored gummed paper may be used in making bar graphs. The work may be done more rapidly, and the finished graph is more effective.

Goals	Materials and References
II. ARITHMETIC (8 weeks)	
A. Increased facility in performing the four fundamental operations involving integers, common fractions, and decimal fractions	
B. Increased ability in finding a per cent of a number with fractional equivalents extended to include: one eighth, three eights, five eighths, seven eighths	MATHEMATICS IN ACTION, Book I, by Hart and Jahn. Pp.133-150. FUNDAMENTAL MATHEMATICS, Book I, by Schorling, Clark, Smith. Unit VIII. World Book Co.
C. The ability to find what per cent one number is of another	
D. An appreciation of the use of percentage in business obtained through the application of percentage to problems in: Profit and loss Discount Commission Simple interest	JUNIOR MATHEMATICS FOR TODAY, Bk. I, by Betz. Chapters X, XIII.

Teaching Suggestions

II. ARITHMETIC

A. Give a standardized diagnostic test in fundamentals to determine the strengths and weaknesses of the pupils. Follow with some remedial work. However, remedial work should continue throughout the semester. Use practice texts found in texts on approved list. See the Handbook for recommendations concerning drill.

B. Review percentage of 7A.

Make the following equivalents automatic: Halves, fourths, eighths.

Through quick oral work show the advantages of using fractional equivalents.

Extend the work in percentage to include finding a given per cent of a number by the formula.

C. In finding what per cent one number is of another, vary the statement of the question as:

4 is what part of 12?
4 equals what per cent of 16?
What per cent of 20 is 5?

Review changing a common fraction to a decimal fraction and a decimal to per cent in connection with this work.

D. Introduce the subject of commission through situations familiar to pupil such as:

Selling magazines, newspapers
Selling Christmas cards
Collecting for newspapers

Point out the advantages and disadvantages of working on a commission.

Select material from the daily newspapers for problems in discount.

Goals	Materials and References
II. ARITHMETIC (Continued)	

Teaching Suggestions

II. ARITHMETIC
(Continued)

Show the advantage of prompt payment of gas and electric bills, and of buying in large quantities.

Since in business practice, interest usually is for short periods of time, 30, 60, or 90 days, use these periods in problems.

Payment of interest is generally made quarterly or semi-annually. Use this fact in problems.

In the application of percentage use many kinds of problems involving finding a per cent of a number and also finding what per cent one number is of another.

Goals	Materials and References
III. MEASUREMENT (2 weeks)	
A. A knowledge of the units used in everyday life for:	STORY OF WEIGHTS AND MEASURES, American Council of Education.
1. Liquid and dry measure.	MATHEMATICS IN ACTION, Bk. I, by Hart and Jahn. Pp. 90-106. LEARNING ARITHMETIC, Bk. 7, by Lennes, Rogers, Travers. Laidlaw Bros., 1942. Pp. 22, 74, 75. MATHEMATICS OF EVERYDAY LIFE by Boyce and Beatty. Inor Publishing Co. Leisure Unit, Pp. 33-48. ARITHMETIC IN GENERAL EDUCATION, Sixteenth Yearbook of National Council of Teachers of Mathematics. Pp. 178-179. Measuring cups and spoons, milk bottles and cartons, pint and quart berry boxes, half bushel and bushel baskets.
2. Measuring weight and time	STORY OF OUR CALENDAR, TELLING TIME THROUGHOUT THE CENTURIES, American Council of Education. WHEN WE SPEAK OF A TENTH, by Christopher Morley, and THE PURSUIT OF ACCURACY, Hamilton Watch Co., Lancaster, Penna.

III. MEASUREMENT

A. Extend the concept of measurement developed in the elementary grades and help pupils acquire added facility in the use of denominate numbers. Remind pupils of the meaning of "denominate."

1-2. Recall the need for standard units of measurement. (See 7A)

Discuss the types of units used in different situations such as:
measuring milk, ice cream, etc. (liquid measure)
measuring tomatoes, peaches, potatoes, grain, etc. (dry measure used for larger quantities, weight used to measure small quantities)
measuring sugar, tea, coffee, coal, etc. (weight)
measuring time: centuries, decades, years, months, days, hours, minutes, and seconds.

Show that counting is exact; measuring is approximate.

Build up tables of measurement. Have pupils bring in sample containers. (See references)

Teach proper abbreviations for units of measure.

Have pupils estimate the weight of objects such as books, blackboard erasers, packages and check with actual weight.

The idea of ratio may be introduced by exercises such as:
4 oz. = ? part of a pound?
What is the ratio of 4 oz. to 1 lb.?

1 qt. = ? part of a gallon?
What is the ratio of 1 qt. to 1 gal.?

Goals	Materials and References
III. MEASUREMENT (continued)	
3. Measuring gas and electricity.	
B. The ability to use these units in significant problems	PROBLEMS IN JUNIOR MATHEMATICS by Gilmartin, Kentopp, Dundon. Newson & Co., 1939. Pp. 21-32.

Teaching Suggestions

III. MEASUREMENT
(Continued)

The story of the development of devices for measuring time is an interesting one. The suggested references will prove helpful to pupils.

3. Provide opportunity for learning to read gas and electric meters. Have pupils bring in bills for study.

B. Give practice in changing from one unit to another, and in performing the four fundamental operations with denominate numbers. Limit work to practical situations.

Have pupils make use of actual time tables in this unit.

Include problems such as:
Finding the number of days between two dates.
This jingle is one way of remembering the number of days in various months:

Thirty days hath September,
April, June, and November,
All the rest have thirty-one
Excepting February alone.
Leap year coming once in four
Gives to February one day more.

Finding the number of years, months, and days between two dates.
Finding a pupil's age in years, months, and days.
Determining the number of children who can be served a cup of milk from a quart of milk, a gallon, etc.
Finding the cost of sending packages of various weights by parcel post.
Increasing and decreasing recipes.

Goals	Materials and References
IV. INFORMAL GEOMETRY (5 weeks)	
A. A knowledge of the use of the compass as a geometric tool, acquired through:	
1. Drawing circles and arcs and studying their properties	EVERYDAY JUNIOR MATHEMATICS, Bk. I, by Betz. Ginn & Co., 1944, Chapter XIII. WORKBOOK IN INTUITIVE GEOMETRY, Betz, Miller. Miller Harter School Supply Co. MATHEMATICS IN ACTION, Bk. I, by Hart & Jahn, Chapter XI and page 204.

Teaching Suggestions

IV. INFORMAL GEOMETRY

A.

1. Discuss the circle in nature, art, and architecture.

   Discuss the contribution of the wheel (circle) to the progress of our civilization. Include the fact that wheels make possible the machines of the modern age.

   Familiarize pupils with the meanings of such terms as radius, radii, diameter, circumference, and arc. Refer to their derivations.

   Emphasize the proper method of holding a compass.

   Practice in using the compass may include drawing circles of different sizes, concentric circles, intersecting circles.

   Lead pupils to discover some of the properties of the circle, such as:
      All radii (and diameters) of a circle are equal.
      A circle has an infinite number of radii (and diameters).
      The diameter of a circle is twice as long as a radius.

   Show that every point on a circle is equidistant from the center. This can be illustrated through the use of string, tape measure, or strips of cardboard.

Goals	Materials and References
IV. INFORMAL GEOMETRY (Continued)	
2. Bisecting lines and angles	
3. Erecting perpendiculars	Compasses Rulers Protractors String (to draw circles) Pictures of circles in Nature, Art, and Architecture
4. Constructing geometric figures such as; rectangles, square, triangles, pentagon, hexagon.	ARITHMETIC WE USE, Bk. 7, by Brueckner, Grossnickle, Merton. Winston Co., 1943. Pp. 164-169, 184-185.
5. Constructing geometric designs	EVERYDAY JUNIOR MATHEMATICS, by Betz. Chapter 13. A RHYTHMIC APPROACH TO MATHEMATICS by E. Somervell (String designs). Available from Miss L. L. Christman, 1217 Elmdale Ave., Chicago.

Teaching Suggestions

IV. INFORMAL GEOMETRY
(Continued)

2. Develop an understanding of the terms "bisect" and "bisector." Show the derivation of the word. (Bi = two; sect = cut). Relate to the words "bicycle," "biped," "biplane."

   Give practice in dividing lines and angles into two equal parts by estimating. Check for accuracy with ruler or protractor. Teach accurate method of bisecting with compasses and straightedge.

   Point out the difference between straight line and straight angle.

3. Use perpendicular lines in various positions to avoid associating of perpendiculars with vertical lines only. Constructing a 45 degree angle and a 60 degree angle without using protractor is a challenge to pupils.

4. Some design work may be introduced here.

5. Decorate paper plates using geometric designs. Color with crayons or water colors.

   Make window transparencies using various geometric designs. Having pupils make designs on paper which is cut the size of the panes in the schoolroom cupboard will provide colorful and interesting coverings for the panes.

Goals	Materials and References
IV. INFORMAL GEOMETRY (Continued)	
B. The ability to find	
1. The circumference and area of a circle.	Cylindrical objects such as cans, wheels, coins, glasses, boxes Circles of various sizes made of cardboard, wood, metal A device for teaching area of a circle: a circle of wood cut into 12 or 16 equal sectors held together by adhesive tape or a metal strip Compasses Rulers Protractors Squared paper

Teaching Suggestions

IV. INFORMAL GEOMETRY
(Continued)

Make snowflakes, based on the hexagon, from thin paper by paper folding and cutting.

Point out symmetry in nature, architecture design, etc.

B.

1. Use the laboratory method in this work with circles.
   a. Have pupils cut a cardboard circle and mark a point on the edge. Starting at this point, roll the circle along the yard stick or on a line drawn on the blackboard until it makes a complete revolution. Measure the distance covered.
   b. Wind a piece of string, a strip of cardboard, or a tape measure around circular objects. Measure the distance across and the distance around. Put the data thus secured in a table. Find the ratio between the circumference and the diameter. Find average of ratios.

To find area of a circle:

a. Cut out a paper circle and fold it through the center. Fold it again and again until it is divided into 8 or 16 equal sectors. Cut along the creases and rearrange the sectors to approximate a parallelogram. Find its area.
b. Use squared paper to show the relation between the area of a circle and the area of a square one side of which is the radius of the circle.

Have pupils report on the history of Pi.

Goals	Materials and References
IV. INFORMAL GEOMETRY (Continued)	
2. The area of a trapezoid	JUNIOR MATHEMATICS FOR TODAY, Bk. I, by Betz. Pp. 374-375, 378-380, 290.
C. An increased understanding of the circumference and area of a circle through the use of significant problems	

IV. INFORMAL GEOMETRY
(Continued)

2. Before taking up the area of a trapezoid, it will be wise to review finding the areas of rectangles, squares, parallelograms and triangles.

   Cut from cardboard or paper two congruent trapezoids. Place them together to form a parallelogram. It is easily seen that the area of one trapezoid is one half the area of the parallelogram thus formed.

C.

Use some practical problems of interest to the pupils which involve finding area and circumference of circles.

8A

COURSE OF STUDY

Goals	Materials and References
I. INFORMAL GEOMETRY (6 weeks)	
A. Familiarity with the various kinds of plane and solid figures	Charts and drawings of geometric figures Models "Geometry for Everyone" by Davis. The Mathematics Teacher, February, 1942. MATHEMATICS IN AVIATION by Osteyee. Macmillan, 1942. Chapter 6.
B. Acquisition of the concept of volume	Boxes made of paper, cardboard, and wood One-inch cubes of wood, paper or soap FUNDAMENTAL MATHEMATICS by Schorling, Clark and Smith. Bk. II. World Book Co. Unit IX.
C. Knowledge of the table of cubic measure with a clear understanding of the meaning of cubic inch and cubic foot	A one-inch cube, cans or other solids which may be filled with sand or water MATHEMATICS IN ACTION, Bk. II, by Hart and Jahn. D. C. Heath. 1939. Chapter 4.

Teaching Suggestions

I. INFORMAL GEOMETRY

A. Review plane and solid figures using models or drawings of figures.

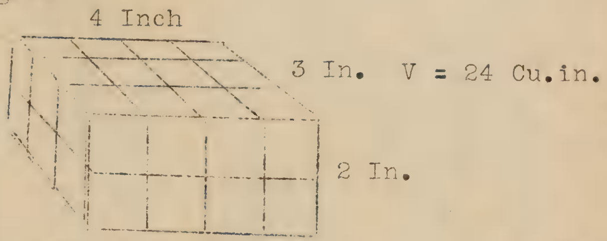
Suggest objects in nature, school room, home and neighborhood that have the same shape as these solid and plane figures.

Have pupils make booklets with drawings and illustrations of plane and solid figures and construct models of solid figures, using squared paper.

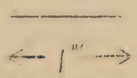
Collect boxes in form of cylinders, prisms, pyramids, cones.

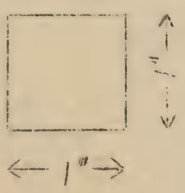
Develop the mathematical concept of a solid.

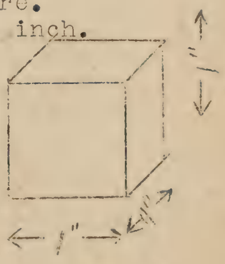
B. Discuss the meaning of volume as the number of cubic units in a solid. Point out that another word for volume is capacity or what a solid holds. Use inch cubes to find the volume of some solid such as a chalk box. Illustrate by means of a diagram.



C. Develop the table of cubic measure. Contrast inch, square inch, cubic inch.







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Goals	Materials and references
I. INFORMAL GEOMETRY (Continued)	
D. Ability to find the volume of the rectangular solid, cube, pyramid, cylinder, and cone	Hollow rectangular prism and pyramid with the same base and height Hollow cylinder and cone with the same base and height (These may be made of metal or paper) Sand. JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Ginn and Co. 1938. Pp 278-279, 288-290. MATHEMATICS IN PRACTICE, Bk. II, by Metzner and Lehr. Franklin Pub. Co. 1936. Pp. 70-76. MATHEMATICS IN THE MODERN WORLD, Bk. II B, by King, Paley and Patterson. Houghton Mifflin Co. 1942. Pp. 282-304.
E. An awareness of the need for finding volumes, acquired by solving significant problems	

Teaching Suggestions

I. INFORMAL GEOMETRY
(Continued)

D. Build a rectangular solid with 1-inch cubes, and develop the rule for finding the volume. Write the rule as a formula. $V = lwh$.

Treat the cube as a special case of rectangular solid. $V = e^3$.

Use a pyramid and a rectangular solid of equal base and height to show that the volume of the pyramid is one-third of the volume of the rectangular solid.

$V = \frac{1}{3} Bh$. Point out that capital "B" stands for the area of the base.

Show that the number of cubic units in the first layer of a cylinder is the same as the number of square units in the area of the base (circle). $V = \pi r^2h$.

Illustrate that the contents of three cones of the same height and base as the cylinder will be needed to fill a cylinder. $V = \frac{1}{3} \pi r^2h$.

E. Keep problems within the experiences of the pupils.

Goals	Materials and References
II. ARITHMETIC (12 weeks)	
A. The ability to make accurate arithmetical computations with ease	THE TEACHING OF SECONDARY MATHEMATICS by Butler and Wren. McGraw-Hill. 1940. Chapter XI Standardized tests Diagnostic tests and drills available in textbooks LEARNING TO COMPUTE (workbook) by Schorling, Clark, Potter, Deady. World Book Co. 1940
B. An increased understanding of the meaning of percentage obtained through: 1. The use of fractional per cents such as: $\frac{1}{2}\%$, $\frac{1}{4}\%$, and those over 100%	Graph blackboard chart Graph paper

Teaching Suggestions

II. ARITHMETIC

A. Give an inventory test in fundamental processes. This should include the four fundamental operations with integers, fractions, and decimals, and also the principal skills relating to percentage and to denominate numbers.

Give specific practice for remedial purposes in written and oral work.

Maintain facility in fundamental processes through drills at regular intervals.

Teach these short cuts:
  (1) Multiplying and dividing by powers of 10
  (2) Using common fractions in place of per cents.

B.

1. Give practice in writing fractions as decimals, as:

   $\frac{1}{2} = .5 \quad \frac{3}{4} = .75 \quad \frac{5}{8} = .625 \quad \frac{1}{4} = .25$

   Contrast a fraction with a fractional per cent such as:

   $\frac{1}{2}$ and $\frac{1}{2}\%$, $\frac{1}{4}$ and $\frac{1}{4}\%$, $\frac{1}{5}$ and $\frac{1}{5}\%$

   Practice changing per cents to decimals, such as:

   $\frac{1}{2}\% = .5\% = .005$

   $\frac{1}{4}\% = .25\% = .0025$

   $\frac{1}{5}\% = .2\% = .002$

Goals	Materials and References
II. ARITHMETIC (Continued)	
2. Familiarity with the process of finding a number when a per cent of it is given	MATHEMATICS IN ACTION, Bk. II, by Hart and Jahn, Chapter V.

Teaching Suggestions

II. ARITHMETIC
(Continued)

Show that $\frac{1}{2}\%$ is $\frac{1}{2}$ of 1%. Use graph chart as shown in 7A.

Show the various methods of finding $\frac{1}{2}\%$ of any number:

a. Find 1% of the number, than take $\frac{1}{2}$ of the result.

b. Change $\frac{1}{2}\%$ to $.001\frac{1}{2}$ and multiply.

c. Change $\frac{1}{2}\%$ to .005 and multiply.

Use a large square to represent 100%. Show that per cents over 100 will not fit in the square.

If 100% = 1, then any per cent over 100% will be more than 1.

Provide drill in moving the decimal point to change from per cent to a decimal. Have pupils state the rule involved in their own words.

2. Review:
   (1) Finding a per cent of a number
   (2) Finding what per cent one number is of another

Introduce finding a number when a per cent of it is given.

Make use of oral and written exercises such as the following:

6 is $\frac{1}{2}$ of _?_   6 is 50% of _?_

4 is $\frac{1}{4}$ of _?_   4 is 25% of _?_

Goals	Materials and References
II. ARITHMETIC (Continued)	
3. The ability to solve problems involving all types of percentage problems	"Crossword Puzzle on Percentage" by Purdy. *The Mathematics Teacher*, March, 1940.
C. Awareness of the personal needs for mathematics developed through:	AN ARITHMETIC FOR TEACHERS by Roantree and Taylor. Macmillan. 1938.
1. Familiarity with the institution of banking to the extent of knowing:	MATHEMATICS FOR JUNIOR HIGH SCHOOL TEACHERS by Schaaf. Johnson Publishing Co. 1931. ARITHMETIC IN GENERAL EDUCATION, Sixteenth Yearbook, National Council of Teachers of Mathematics.

II. ARITHMETIC
(Continued)

Develop the formula $B = \frac{P}{R}$ for finding a number when a per cent of it is given.

Remind the pupil that the base represents the whole or 100%.

Apply formula to exercises involving the use of per cents as fractions and decimals.

Include problems such as:
(1) How much money must be invested to obtain a certain amount of interest?
(2) How much must sales amount to in order to earn a certain commission?

3. This unit will provide an opportunity for pupils to apply what they have learned about percentage in:
Discounts (single and successive)
Commission
Interest

C.

1. Banking

Trace the history of banking.

Discuss the services rendered by banks.

Dramatize the opening of a bank account. Characters: Teller, bank officials, person opening account, and person introducing new customer.

Point out the difference between a checking account and a savings account.

Goals	Materials and References
II. ARITHMETIC (Continued)	
a. Services rendered by a bank	Bank forms available at a bank. Bank forms available at school: Signature cards Deposit slips Withdrawal slips Pass books Checks Promissory notes
b. How to open an account	
c. How to use a checking account	
d. Difference between a checking account and a saving account	JUNIOR MATHEMATICS FOR TODAY, Bk II, by Betz. Pp. 55-88.
e. How to write checks properly	
f. How to compute interest	
g. How to borrow money, including the use of the promissory note	

Teaching Suggestions

II. ARITHMETIC
(Continued)

Give practice in filling out bank forms: signature cards, deposit slips. Show the use of bank book and bank statement.

Discuss checks and their uses.

Trace travels of a check from the time it is written until it is cancelled.

Have pupils make out checks and check stubs.

Emphasize the following in the study of bank checks:

- Always use ink in writing a check.
- Write the amount in figures and words.
- Use the signature that is on the signature card filed at the bank.
- Point out to pupils the value of adopting a signature to be used on all of their written work.
- Never make changes on a check.
- Show the different ways of indorsing checks:
    - Blank indorsement (John A. Brown)
    - Special indorsement (Pay to the order of Mary Smith, John A. Brown)
    - Restricted indorsement (For deposit only, John A. Brown)

Review interest formula, i = prt.

Calculate interest by decimals, by the cancellation method, and from tables.

Use periods of time common to business:
annually, semi-annually, quarterly and 30, 60, 90 days.

Goals	Materials and References
II. ARITHMETIC (Continued)	
2. An understanding of the purposes and kinds of taxes obtained by solving problems involving real estate, income, sales taxes	Tax bills if available Income tax forms Articles from newspapers SOCIAL UTILITY ARITHMETICS, Bk. VI, by Strayer-Upton. Pp. 473-512. ARITHMETIC WE USE, Bk. VIII, by Brueckner, Grossnickle, Merton, Winston, 1942. Chapter VIII. LEARNING ARITHMETIC, Bk. VIII by Lennes-Rogers-Travers-Laidlaw. 1942. Pp. 184-200. SECOND COURSE IN THE NEW MATHEMATICS by Edgerton and Carpenter. Allyn and Bacon. 1939. Unit X. MATHEMATICS FOR JUNIOR HIGH SCHOOL TEACHERS by Schaaf. Pp. 165-178.

Teaching Suggestions

II. ARITHMETIC
(Continued)

Have pupils fill out promissory notes. Give practice in finding interest, date of maturity, maturity value. Emphasize the responsibility involved in indorsing notes. Pupils may make a booklet on banking including the above topics.

2. Taxes

Discuss the necessity of taxation: why we pay taxes, how much we get for what we pay in taxes, the cost of government, kinds of taxes and how revenues used in financing government enterprises are obtained. Include poll tax, tolls, excise tax, and customs or duties for information. Distinguish between national, state, and local taxes. Bring out the fact that everyone is a taxpayer.

Show that when taxes increase, except in war time, it is largely because of increasing demands made by the citizens in the interest of public welfare and there are new services as a result.

Show that many of the benefits which we enjoy in a modern community are made possible only through taxation.

Consider the way city taxes are levied, based on the budget, assessed valuation of property, tax rate.

Discuss the different methods of expressing the tax rate on property and show how to change from one to the other: mills per dollar, cents per dollar, dollars per hundred dollars, dollars per thousand dollars, and as a per cent. Apply to simple practical problems. In computing real estate taxes use the current D. C. tax rate.

Goals	Materials and References
II. ARITHMETIC (Continued)	
3. An understanding of the importance of thrift acquired through:	Materials brought by pupils on earning and spending money. Budget forms, graphs, and articles found in magazines and newspapers.
a. Studying and making out budgets of time and money, both family and personal	MATHEMATICS IN LIFE by Schorling and Clark. World Book Co. 1937. Unit Vl.
b. Familiarity with the national and city budget,	MATHEMATICS AND LIFE, Bk. I, by Ruch, Knight, Studebaker. Scott Foresman. 1937. Pp. 129-140.
c. A study of savings and investments	Booklets published by insurance companies on budgets. THE SECRET OF THRIFT by C. B. Upton. Savings Bank Division, American Bankers Association, 22 East 40 Street, New York.

II. ARITHMETIC
(Continued)

Give practice in computing income taxes when the gross income and exemptions are known.

Have pupils draw a circle graph showing how the tax dollar is distributed in this city.

Interpret graphs and statements concerning taxes from newspapers and magazines.

3. Thrift

Discuss the need for thrift in the use of time and money. Have pupils collect sayings of famous men concerning thrift. Point out the advantages of budgeting time and money. Consider the school day as an example of budgeting time.
Graph: (1) How I Spend My Day
(2) How I Spend My Allowance

Discuss with pupils the meaning of keeping within a budget. Make simple budgets of family incomes. Select average incomes of various amounts. This topic offers an opportunity for making circle graphs. Point out the importance of keeping accounts of receipts and expenditures.

Have pupils bring in articles on national and city budgets for class discussion.

The study of investments may well include: the reading of the stock page in the newspapers; the dramatization of the organization of a stock company with the issuing of stocks and bonds; the dramatization of the purchase of securities.

Emphasize the qualities of a good investment, safety, yield, salability.

Goals	Materials and References
II. ARITHMETIC (Continued)	
4. A study of insurance	Literature circulated by insurance companies. (Pamphlets, posters, statistics, tables)
a. The value of insurance as a cooperative undertaking for mutual protection	Newspaper and magazine articles JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Pp. 217-235.
b. The value of various types of insurance obtained through the solution of significant problems involving personal and property insurance	SECOND COURSE IN THE NEW MATHEMATICS by Edgerton and Carpenter. Pp. 257-277. MATHEMATICS IN DAILY USE by Hart et al. Pp. 226-241. ARITHMETIC WE USE, Bk. VIII, by Brueckner, Grossnickle, Merton. MATHEMATICS FOR TODAY, Bk. II, by Betz. Pp. 123-131, 328-333. THE ROMANCE OF LLOYDS by Worsley and Griffith. Holman-Curl Inc. FUNDAMENTAL MATHEMATICS, Bk. II, by Schorling, et. al. Unit VIII.

II. ARITHMETIC
(Continued)

One possibility in dramatizing the formation of a corporation is to suppose that a member of the class has invented something unique and does not have money to manufacture it. The pupils can then "form a corporation," invest money, buy stocks or bonds and thus learn many lessons about investments.

4. Insurance

Discuss the meaning and importance of insurance in modern life.

Include:

a. The risks and uncertainties involving life and property.

b. The losses sustained by individuals before the days of insurance.

c. The situations in which insurance may be compulsory or highly desirable. (Mortgages, automobile liability insurance, parcel post shipments, installment purchases).

Have pupils list the kinds of insurance carried by their families.

Provide sample copies of insurance policies for pupils to examine.

Discuss the various types of personal insurance, such as life, health, accident, old age, and group hospitalization.

Goals	Materials and References
II. ARITHMETIC (Continued)	

Teaching Suggestions

II. ARITHMETIC
(Continued)

Emphasize the two-fold purpose of some types of life insurance, that of protection for dependents and investment for the insured.

Safety and accident prevention may be correlated with the study of accident insurance. Material may be secured from the A. A. A.

Have pupils list the kinds of insurance that a property owner may carry on his property.

Pupils may report on interesting kinds of insurance. Lloyds of London affords interesting material for such reports.

When discussing fire insurance stress the importance of having an inventory of household goods. Point out the annual loss by fire due to carelessness.

In this age of automobiles the various types of automobile insurance should be discussed with special emphasis on liability and property damage.

Acquaint pupils with special terms commonly used in the insurance field, such as; policy, premium, rate of insurance, face of policy, dividend, beneficiary.

Discuss factors which determine the premium rate for a given kind of policy.

Use problems involving the finding of premiums for different types of insurance.

8B

COURSE OF STUDY

Goals	Materials and References
I. (1 wk) An appreciation of the importance of symbolism in modern life and the essential part it plays in mathematics	ALGEBRA, AN INTERESTING LANGUAGE by Breslich. Newson. 1939. ORIGINS OF MATHEMATICS, 16 mm. Sound Film. JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Ginn and Co. 1938. Pp. 239-243.
II. (4 wks) An understanding of the importance and use of the formula as a mathematical tool for expressing relationships acquired by: A. Developing algebraically the formulas for perimeter, area, and volume	MATHEMATICS IN ACTION, Bk. II, by Hart and Jahn. D. C. Heath Co. 1939. Pp. 55-77. MATHEMATICS FOR TODAY, Bk. II, by Douglass and Kinney. Holt and Co. 1940. Pp. 215-234. FUNDAMENTAL MATHEMATICS by Schorling, Clark, Smith. World Book Co. 1944. Pp. 305-312. "Symbolism" MATHEMATICS IN GENERAL EDUCATION. P. E. A. Appleton-Century. 1940.

Teaching Suggestions

I. Introduce symbolism by a discussion of signs and symbols used in everyday life. Consider:
$, ¢, &, #, %, c/o.

Stress the fact that numerals are symbols that stand for numbers. Explain that "2" is no more a number than "x" is. Both are symbols that represent quantities. Consider number symbols that have been developed by other civilizations.

II.

A. Extend the pupils' concept of formulas through a discussion of chemical formulas, pharmaceutical formulas, mathematical formulas, recipes in cook books, and so-called secret formulas.

Stress the fact that a mathematical formula is a rule expressed in symbols.

Review the following formulas (see also 7A, 7B, and 8A) by developing them algebraically:
Perimeter: rectangle, square, triangle
Circumference: circle
Area: rectangle, square, triangle, circle
Volume: rectangular solid, cube, pyramid, cylinder, cone

Have pupils develop formulas in exercises such as the following:
Find the perimeter of an isosceles triangle.
Find the perimeter of an equilateral triangle.
Find the perimeter of a rectangle whose base is three times as long as its altitude.

Goals	Materials and References
B. Evaluating formulas	
C. Graphing Formulas	
D. Writing formulas from data in tables	JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Pp. 307-313.
E. Using formulas in solving significant problems	MATHEMATICS IN ACTION, Bk II, by Hart and Jahn. Chapter IV.
III. (1 wk) An understanding of the language of algebra, obtained through:	ALGEBRA: A WAY OF THINKING by Mitchell & Walker. Harcourt, Brace Co. 1936. Chapter I.
A. Evaluating simple algebraic expressions	

B. Practice in using formulas offers excellent opportunity for maintaining skill in the fundamental processes and for problem solving.

The evaluation work can include whole numbers, fractions, or decimals. Stress those in which most practice is needed.

C. Graphing a formula offers another opportunity to show how related values obtained from the formula depend on each other.

D. The ability to see number relationships is of value in all phases of mathematics. Our scientific age requires a great deal of experimentation, the results of which can often be expressed by means of a formula.

Discuss with pupils experiments which involve the relationships of quantities such as those resulting in the making of soaps, synthetic rubber, plastics, synthetic gasoline, chemicals, recipes, time-saving formulas in industry, airplane structure involving lift and speed, etc.

Use material found in textbooks to give practice in making formulas based on tables.

E. Make use of many problems to have pupils realize the many applications of formulas, including formulas from science, business, shops, etc., such as:

$C = \frac{5}{9}(F - 32)$ $i = prt$ $d = rt$ $P = RB$

III.

A. Give much practice in reading and explaining the meaning of algebraic expressions such as:

$3a$ means 3 times a
$2xy$ means 2 times x times y
$\frac{a}{2}$ means a divided by 2, or $\frac{1}{2}$ of a
$\frac{abc}{3}$ means a times b times c divided by 3

Goals	Materials and References
III. (Continued)	
B. Expressing statements by means of signs, letters, and numerals.	ALGEBRA IN EASY STEPS by Stein. Newson and Co. New York. 1942.

Teaching Suggestions

III. (Continued)

$4x + 5y + 8z$ means 4 times x plus 5 times y plus 8 times z.
y^2 means y times y.
x^3 means x times x times x.
a^4 means a times a times a times a, and is read, "a raised to the fourth power."

Introduce the word "exponent." Develop the meaning of a positive integral exponent as the number of times the base is to be used as a factor.

Have pupils evaluate monomials orally at first to insure understanding. Use the same expression 2 or 3 times with different values for the unknowns in order to show that its value depends upon the values of the unknowns.

Point out that since the letters in an algebraic expression stand for numbers, every algebraic expression represents a number. The value of the expression depends upon the values of the unknowns.

Give practice in evaluating polynomials, having pupils evaluate each term in order. Make use of expressions involving exponents to insure an understanding of their meaning.

Evaluating algebraic expressions offers an excellent opportunity for reviewing the fundamental operations. Assign fractional and decimal values as well as integral ones.

B. Point out the fact that translating word statements into algebraic expressions is very important in understanding the language of algebra, and in solving problems later on.

	Goals	Materials and References
IV. (4 wks)	A working knowledge of the equation obtained through: A. A familiarity with the principles underlying an equation	FUNDAMENTAL MATHEMATICS, Bk. II by Schorling et al. Pp. 116-131. ALGEBRA, AN INTERESTING LANGUAGE by Breslich. Pp. 46-49. "Classroom Practice in the teaching of Everyday Mathematics" by R. J. Mejdak, The Mathematics Teacher, December, 1941.

Teaching Suggestions

III. (Continued)

Make use of statements involving the sum, difference, product, and quotient of two numbers to insure a knowledge of the operations associated with them. Include statements involving such expressions as "increased by," "decreased by," "diminished by," "more than," "less than," "3 more than," "3 times as much," "exceeds."

IV.

A. Ascertain the pupils' knowledge of the meaning of equations. What word does the word "equation" bring to mind?

Point out that the equation is one of the most useful "tools" algebra has contributed to mathematics. By means of it, we are able to solve problems which would be very difficult to solve by arithmetic.

Point out how an equation differs from a formula.

Discuss a set of equations such as the following from the standpoint of how to read them and what they mean:

(1) $4x = 12$

(2) $\frac{x}{9} = 4$

(3) $x - 7 = 3$

$x + 9 = 15$

$9 + x = 12$

The following game of "mind reading" can be used to stimulate interest:

Teacher: Think of a number, double it, divide it by the original number, add 5. (The result is always 7). (Why?)

Goals	Materials and References
IV. (Continued)	
	A pair of scales or balances borrowed from the science department
	A balance constructed by pupils

Teaching Suggestions

IV. (Continued)

Introduce the expression "Solving an equation." Like solving a mystery it means finding that which is unknown. The letter stands for the unknown.
Have pupils solve some equations by inspection.

Determine the correct answer to each equation by checking all answers given by the class. Insist on having pupils give answers in full, as x = 4. Point out that the one that checks is the one that is said to satisfy the equation. Pupils should then see the need for checking, since the answer they choose is not always correct.

Introducing a few more difficult equations at this point will show the pupils a need for a better method of solving equations than "inspection".

In illustrating the principles underlying equations, a pair of old-fashioned scales are helpful. Have pupils observe that if the scales are balanced to begin with, then no matter what one does to the quantities on the left (mathematically) the scales can be balanced by doing the same thing to the quantities on the right.

These principles can be combined into the following single statement called the Rule of Equality: If equals are increased or decreased or multiplied or divided by equals, the results are equal.

Solve by axioms. Do not teach transposition.

Goals	Materials and References
IV. (Continued)	
B. Solving and checking equations of the four basic types	JUNIOR MATHEMATICS FOR TODAY, Bk II, by Betz. Pp. 315-341.

IV. (Continued)

B. Point out the four fundamental types of equations:

(1) $4x = 36$ (3) $x + 3 = 7$
(2) $\frac{x}{7} = 3$ (4) $x - 4 = 9$

Tabulate results for further discussion:

Equation	Operation indicated in the equation	Operation to be used in solving the equation	What to do
$4x = 36$	×	÷	Divide both members by 4
$\frac{x}{7} = 3$	÷	×	Multiply both members by 7
$x + 3 = 7$	+	−	Subtract 3 from both members
$x - 4 = 9$	−	+	Add 4 to both members

Give pupils practice in determining what operation to use until they can make the following generalizations:

(1) Each operation used gets rid of some unwanted number.
(2) The operation to be used in getting rid of a number is always the opposite of the operation shown as illustrated in the above table.

Stress form and checking in solving equations. Pupils often miss the significance of checking. Two methods of checking are suggested:

To check the root $x = 4$ in the equation $3x - 5 = 7$:

Does $3 \cdot 4 - 5 = 7$? or $3 \cdot 4 - 5 \overset{?}{=} 7$
Yes $7 = 7$

Goals	Materials and References
IV. (Continued)	
C. Writing equations for word statements expressing quantitative relationships	
D. Using equations in solving simple problems	MATHEMATICS IN ACTION, Bk. II, by Hart and Jahn. Chapter X.

Teaching Suggestions

IV. (Continued)

C. Precede the use of the equation in problem solving by giving practice in translating word statements into algebraic expressions. Begin with simple statements such as:

Three times a certain number is 45. ($3x = 45$)
The perimeter of a square is 16 inches.
($P = 16$ or $4s = 16$)
One half of a number is 18.
($\frac{1}{2}n = 18$ or $\frac{n}{2} = 18$)

D. Have pupils follow a definite procedure in solving problems such as:

(1) Determine from the question what is to be found.
(2) Let "x" (or any other letter) stand for this unknown.
(3) Draw a figure if possible.
(4) Develop the proper equation.
(5) Solve the equation by using the axioms.
(6) Check the answer in the original problem, not in the equation that was set up.

Choose problems that come within the experiences of the pupils. There is an opportunity here for correlation and review by showing how problems in the three cases of percentage could be solved algebraically.

Problems must be kept very simple if only the four basic types of equations are taught. The field of application becomes much broader if equations involving more than one step are taught.

Goals	Materials and References
V. Familiarity with squares (1 wk) and square roots and the ability to find these by the use of tables	SECOND COURSE IN THE NEW MATHEMATICS by Edgerton and Carpenter. Allyn and Bacon. 1934. Unit V. MATHEMATICS IN PRACTICE, Bk. II, by Metzner & Lehr, Franklin Pub. Co. 1936. Chapter III.

Teaching Suggestions

V. Discuss the meaning of "square":
  - a. as a geometric form (noun)
  - b. as the process of multiplying a number by itself (verb)
  - c. as the product of two equal factors (noun)

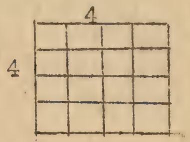
Review the use of the exponent "2." Have pupils make the table of squares from 1^2 to 25^2 for a clear understanding of the process of squaring. Point out its relationship to the multiplication tables. (A square is the product of two equal factors). Have pupils memorize table for future use; grouping according to difficulty will aid in this.

The following relationships will give pupils a glimpse of some of the interesting facts about squares:

$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
etc.

Extend the process of squaring to fractions, decimals, and mixed numbers.

Show the relationship between the side of a square and its area.



$$A = 4 \times 4$$
$$A = 4^2\text{, or } 16$$

Give practice in the use of table for finding the squares of numbers larger than 25.

Review work with formulas by solving problems involving squares, such as the following:

$$A = s^2 \qquad A = \pi r^2 \qquad V = \pi r^2 h$$
$$V = \frac{1}{3} \pi r^2 h$$

Emphasize that the square root of a number is one of two equal factors of the number.

Goals	Materials and References
VI. (1 wk) Familiarity with the basic concepts of ratio and proportion.	SOCIAL UTILITY ARITHMETICS, Bk. VI, by Strayer-Upton. American Book Co. 1944. Chapter 26. JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Pp. 147-158, 359-360. "Ratio in Music"; "Ratio in Geography" from MATHEMATICS IN MODERN LIFE by McCormack. D. Appleton Century. 1937. Pp. 278-281, 291 "A Mathematics Class Takes Music Lessons" by L. J. Cronbach. *The Mathematics Teacher*, January, 1943.

Teaching Suggestions

V. (Continued)

Introduce the radical sign ($\sqrt{}$) as a symbol for square root.

Use facts from the table of squares to insure understanding of the meaning of square root. Much oral work can be done. Do not overlook the square roots of fractions such as: $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{9}{16}$, etc.

Give practice in:
  a. estimating square roots.
  b. finding square roots by the use of tables.

Make applications to problems involving the process of finding the side of a square when the area is known.

See VII for further suggestions.

VI. Introduce ratio as one of several ways of comparing quantities. It may be written as a fraction or with the symbol(:). The fractional form is preferred.

Remind pupils of the principle that a fraction remains unchanged if both numerator and denominator are multiplied or divided by the same quantity.

Have pupils bring material from magazines and newspapers showing the use of ratios.

Some examples of the different uses of ratio:

(1) If there are 40 pupils in a section, of which 15 are boys and 25 girls, consider the ratio of the number of boys to the total, girls to the total, and boys to girls or girls to boys.

(2) The "batting average" of a player is the ratio of the number of hits to the number of "times at bat."

(3) "10 carat gold" means 10 parts of gold out of 24; 14 carat means 14 out of 24.

Goals	Materials and References
VI. (Continued)	ARITHMETIC WE USE, Bk. VIII, by Brueckner, Grossnickle, Merton. Winston. 1943. Pp. 248-254.

VI. (Continued)

Stress the fact that the two numbers compared must be expressed in the same unit.

Give practice in comparing quantities by means of ratios. This affords opportunity to review denominate numbers.

Show how the housewife uses ratios in increasing and decreasing quantities in recipes.

The Greeks considered five-eighths as the ratio between the width and the length of a rectangle which is most pleasing to the eye. They called it "The Divine Proportion." It was used in building temples.

Introduce proportion as a statement of equal ratios.

In writing proportions, the symbols (::) is no longer used.

Familiarize pupils with the meanings of such terms as: cross products, means, and extremes. Lead pupils to discover the rule of proportion, namely, "The product of the means equals the product of the extremes" or "The cross products are equal."

Make use of these rules in finding missing terms in proportions. In checking use a different type of process than was used to obtain the answer.

Have pupils solve problems by means of proportions. See VII for further suggestions.

Teaching Suggestions

VII. Discuss the need man has found for measuring distances indirectly. Discuss measurement of the following: diagonals of rooms, distances across lakes and rivers, distances to sun and other planets, heights of mountains, distances and lengths in astronomy, navigation, warfare.

Discuss the following methods of indirect measurement to be studied:

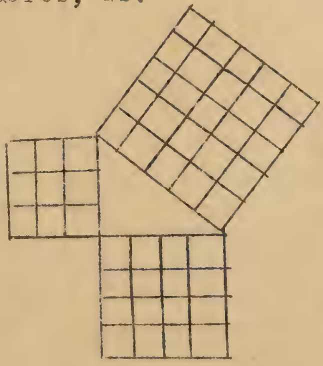
(1) Pythagorean Theorem
(2) Similar Figures
(3) Scale Drawing

A. Review the meaning of right angle and right triangle.

Include a brief history of the part the right triangle has played in mathematics, with emphasis on the Egyptian rope stretchers, Pythagoras, and the fact that many proofs of the Pythagorean Theorem have been developed by other mathematicians.

Introduce the term "hypotenuse," and give pupils practice in picking out the hypotenuse in right triangles drawn in different positions.

Help pupils discover for themselves the Pythagorean Rule by actually drawing a 3-4-5 triangle with a square on each side and then counting the squares, as:



Goals	Materials and References
VII. (Continued)	
B. Similar Figures	LEARNING ARITHMETIC, Bk. VIII, by Lennes et. al. Pp. 222-227.

VII. (Continued)

Other "Pythagorean numbers" that may be used for sides are 5-12-13 or 8-15-17 and any triples of numbers in these ratios.

B. Develop the concept of similarity. Consider the different sizes in hats, shoes, dresses, suits, dishes, pans, tools, pictures, buttons, jewelry.

Have pupils enlarge and reduce designs. This can be done very easily by using squared paper. Also mention the use of "proportional dividers" and the pantograph.

Lead pupils to discover the following properties of similar triangles:

(1) They have the same shape but not the same size.
(2) Corresponding angles are equal.
(3) Corresponding sides are in proportion.

The word "corresponding" needs to be taught carefully.
Give pupils practice in selecting corresponding sides and angles.

Give practice in finding unknown sides of similar triangles by the use of proportion.

Make applications in problems involving the use of similar figures to find unknown lengths.
Consider:
(1) Determining the height of a tree, flagpole, or building by shadow measurement.
(2) Enlarging or reducing pictures and diagrams.
(3) Finding the lengths of ponds or lakes.
Show how Boy Scouts use indirect measurement in estimating distances and heights.

Goals	Material and References
VII. (Continued)	
C. Scale Drawing	Road maps of D. C. and vicinity Maps in geography and history Blueprints MATHEMATICS IN LIFE by Schorling and Clark. Unit III. MATHEMATICS FOR AIR CREW TRAINEES. TM 1-900, War Department, Washington, D. C. Section IV. BLUEPRINT READING AT WORK by Rogers and Welton. Silver Burdett. 1942.
D. Problems involving indirect measurement	ALGEBRA, FIRST COURSE by Breslich. Laidlaw Brother. 1939. Chapter XVII.

Teaching Suggestions

VII. (Continued)

C. Point out the need for scale drawing in making diagrams, maps, blueprints, pictures, etc. where it is not possible, to draw a full-size picture of an object. Consider the very few cases where full-size pictures are used. Also consider the work of architects, surveyors, and draftsmen.

Explain meaning of fractions printed beneath pictures of birds, animals, and fish, in the dictionary.

Emphasize the necessity for accurate measurement in drawing to scale.

Give practice in reading and making scale drawings. Finding straight-line distances between places on a map is interesting because of the great amount of air travel. Use road maps, geography and history maps, and blueprints.

Begin with lines, simple scales, and simple diagrams such as, rectangles, squares, triangles, tennis courts, baseball diamonds, football fields, rooms, bookcases, tables.

Then show how scale drawing may be used to determine unknown lengths. Discuss the degree of accuracy with which this can be done.

D. Develop an appreciation of the relative values of the three methods of indirect measurement studied by having pupils work several problems all three ways.

Use many problems within the pupils' range of experience.

Goals	Materials and References
VIII. (2 wks) A knowledge of signed numbers as a part of the number system, obtained through:	MATHEMATICS FOR JUNIOR HIGH SCHOOL TEACHERS by W. L. Schaaf. Johnson Publishing Co. 1931. Pp. 315-323.
A. An appreciation of the need for negative numbers	JUNIOR MATHEMATICS FOR TODAY, Bk. II, by Betz. Chapter XVI. TEACHING OF SECONDARY MATHEMATICS by Butler & Wren. McGraw-Hill. 1941. Pp. 293-300.

Teaching Suggestions

VIII.

A. Point out the lack of sufficient numbers in the number system as it is known by the pupils up to this point to provide answers for many mathematical problems. Consider the following: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11..........
(all common and decimal fractions included between 0 and 1)

Subtract:

$$\begin{array}{cccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 4 & 3 & 2 & 1 & 0 & ? & ? & ? \end{array}$$

Bring out the fact that the values of some of these answers must be less than zero; namely, one less, two less, and three less, respectively. Explain that negative numbers are the symbols mathematicians invented to express such quantities.

Point out the relationship of these numbers to those with which the pupils are already familiar, the positive numbers. Show that "0" becomes the point of reference for this number system. ...-7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, ...

Consider the following questions with pupils:
Do these numbers have meaning?
Have they application in life?
Can the four fundamental operations be performed with them?

Goals	Materials and References
VIII. (Continued)	
B. Familiarity with the uses of the signed numbers	MATHEMATICS IN THE MODERN WORLD, Bk. 2B, by King et al. Chapter II. PRACTICAL MATHEMATICS, Bk. IV, by Antonville and Trube. Noble and Noble. 1942. Pp. 229-259.

VIII. (Continued)

B. Have pupils name situations where quantities more and less than zero, or above and below zero are involved.

Consider:

- Thermometers - temperature
- Game scores (card games and others)
- Battery charges
- Locations with respect to sea level
- Position of an airplane as contrasted with the position of a submarine
- A person's financial status, in debt or out of debt
- Marking shipments of goods, overweight or underweight
- North and south latitude
- East and west longitude
- Significance of equator and prime meridian

Discuss the significance of zero:

(1) as referring to quantity.
(2) as a point of reference.

Consider questions of the following types:

(1) What is the significance of the zero point on the thermometer? Does it mean there is no temperature at that point?
(2) Does below sea level necessarily mean underwater?
(3) How could the financial status of each of the following men be expressed:
    a. one who owes nothing and has nothing
    b. one who has $5.00 and owes $85.00
    c. one who has $50.00 and owes $10.00
    d. one who has $25.00 and owes $25.00

Bring out the fact that positive and negative numbers are used to represent opposites.

Goals	Materials and References
VIII. (Continued)	
C. The ability to perform four fundamental operations with signed numbers and monomials.	

Teaching Suggestions

VIII. (Continued)

Stress the difference between a signed number and its absolute (or numerical) value.

Point out that the plus and minus signs have two meanings:
(1) to indicate addition, subtraction.
(2) to indicate kind of number.

Refer to signed numbers as positive and negative rather than plus and minus numbers.

Use horizontal and vertical number scales on a graph chart. Have each pupil construct a number scale on graph paper for his own use.

C. Keep in mind that developing the fundamental operations with signed numbers offers an excellent opportunity for increasing the pupil's understanding of the meaning of each operation.

Addition
Develop the definition for addition as the process of combining quantities.

Make use of pupils' experiences with games where points can be gained or lost. Find final scores for groups of 4 or 5 players (use small numbers to begin). Have pupils pick out the winners.

Check results by using the number scale.

Give practice in finding totals for columns of signed numbers, requiring pupils to combine numbers in order as they appear. Then have pupils find totals involving larger numbers.

Help pupils discover for themselves the rule for adding signed numbers.

Goals	Materials and References
VIII. (Continued)	"Teaching the Subtraction of Signed Numbers" by A. E. Katra. *The Mathematics Teacher*, November, 1940. "Teaching the Subtraction of Signed Numbers" by M. G. Ecke. *The Mathematics Teacher*, February 1941.

VIII. (Continued)

It is very important that pupils master the process of adding signed numbers since all the other operations are related to it.

Use the horizontal as well as the vertical forms in addition.

Work with monomials will add to the interest and give extra practice.

Subtraction

Develop the definition of subtraction as the process of finding the difference.

Point out that there is a difference between the values of **any** two numbers on the number scale; therefore, it should be possible to subtract any two numbers.

Work with pupils, orally at first, in finding the difference between numbers. Finding the difference between temperatures works well if the absolute values are kept small.

Point out that in algebra just as in arithmetic the minuend is to be compared with the subtrahend.

Give many examples in finding the difference, using the number scale or the thermometer to check results whenever necessary.

Have pupils work out sets of exercises with larger numbers.

Develop with pupils the rule for subtracting signed numbers and give practice in using it.

Include the subtraction of monomials.

Goals	Materials and References
VIII. (Continued)	"The Laws of Signs in Multiplication" by J. H. Zant. The Mathematics Teacher, April, 1939.

VIII. (Continued)

Multiplication

Develop the meaning of multiplication as the process of repeated addition.

Review the different ways of indicating multiplication.

Point out that since only two numbers can be multiplied together at one time, only the four following types can possibly be encountered:

(a): (b): (c): (d):
(+4)(+3) (+4)(−3) (−4)(+3) (−4)(−3)

The following interpretations of these exercises may prove helpful:

(a) means add "+3" four times. Compare this with the meaning of 4×3.
(b) means add "−3" four times.
(c) means subtract "+3" four times. Review rule for subtraction. Begin with 0.
(d) means subtract "−3" four times.

As a result of (c) and (d) the definition of multiplication should be extended to include repeated subtraction.

Have pupils study the results of these exercises and make up the rule governing multiplication of signed numbers.

Give practice in multiplying by use of the rule. Include exercises involving more than two numbers, such as (-5) (−2) (−3). Pupils sometimes forget that they must multiply two at a time to determine the correct sign for the answer. This will avoid the mistake of giving +30 for the answer, even though the signs are all alike.

Include the multiplication of monomials.

Goals	Materials and References
VIII. (Continued)	

VIII. (Continued)

Division
Develop the concept of division as the inverse of multiplication.

Formulate the rule for dividing signed numbers and give practice in dividing by using the rule. Include the division of monomials.

Call the attention of pupils to the fact that a knowledge of signed numbers is very important for further work in mathematics.

ELEMENTARY ALGEBRA 1
(First Semester)

ELEMENTARY ALGEBRA 1
(First Semester)

Goals	Materials and References
I. An appreciation of the symbolic nature of algebra based on an understanding of: (2 weeks)	Origins of Mathematics - 16 MM. SOUND FILM. Visual Education Department, D. C. Public Schools. CHART: "Highlighted History of Algebra". Scott, Foresman Co.
A. The use of algebraic expressions to represent numbers	"The Teaching of Algebra in the Junior High School". THE TEACHING OF SECONDARY MATHEMATICS by Butler and Wren. McGraw-Hill. 1941. Pp. 275. HISTORY OF MATHEMATICS, Volume II, by D. E. Smith. Ginn & Co. 1925. Pp. 378-393. "Historical Note on the Development of Algebra". ALGEBRA FOR TODAY by W. Betz. Ginn & Co. 1937. Pp. 8. ALGEBRA THE LANGUAGE OF MATHEMATICS by J. R. Overman. Lyons & Carnahan. 1940. Historical Notes on Development of Algebra. FIRST YEAR ALGEBRA by Schorling, Clark, Smith. World Book Co. 1943. Chapters 1 and 2. Pp. 37, 104, 142, 246. ESSENTIALS OF ALGEBRA, First Course, by Hart. D. C. Heath & Co. 1941. Pp. 62. Chapter 2.

ELEMENTARY ALGEBRA 1
(First Semester)

Teaching Suggestions

I. Keep in mind the "Purposes of Teaching Mathematics," as stated in the HANDBOOK, particularly section IV. For present recommended usage in terms and symbols, see the Appendix.

Make use of historical notes on the development of algebra. They will motivate the subject and foster better understanding.

A. It will be necessary to test the class carefully to find which of the beginning concepts of algebra have already been mastered.

Introduce algebra as generalized arithmetic.

Point out the nature of written symbols. Do not give the impression that letters are abbreviations. Avoid teaching the idea, for instance, that $2a + 3a = 5a$, because 2 apples + 3 apples equals 5 apples.

Since it will take time to develop the understanding of the nature of symbolism, give varied experience with letters as general numbers in order to establish the concept.

Develop the meaning of such statements as:

$x \cdot 0 = 0$ $\quad x - x = 0$

$1 \cdot x = x$ $\quad xy = yx$

$x + y = y + x$

Recall the fact that multiplication is a short method of addition. ($a + a + a + a + a = 5a$)

Teachers Manuals to accompany algebra text books are available upon request to the publishers. They contain many excellent suggestions which teachers will find helpful.

ELEMENTARY ALGEBRA 1
(First Semester)

Goals	Materials and References
I. (Continued)	
B. An appreciation of the formula and equation as means of expressing rules and quantitative relationships.	ALGEBRA FOR TODAY by Betz. Ginn & Co. 1937. Chapter II. MATHEMATICS IN AVIATION by Osteyee. Macmillan Co. 1942. Chapter 17 ELEMENTARY ALGEBRA by Edgerton and Carpenter. Allyn & Bacon. 1939. MATHEMATICS FOR AIR CREW TRAINEES, TM 1-900. War Department. 1943. AVIATION MATHEMATICS by Felker, Paine, Schweinhagen. Bruce Publishing Co. 1944. Pp. 107, 110-114. "Teaching the Laws of Algebra or Mathematics is a Language" by C. Denbow. *The Mathematics Teacher*, December 1941. Balance scales can be borrowed from the science department.

I. (Continued)

B. Use formulas with which the pupils are already familiar.

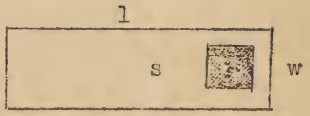
Emphasize again the fact that each letter in a formula is a general number, not the abbreviation of a word.

Evaluation must be taught with care, Explain every step of the process. In "substituting" show that the use of the word is the same as that of replacing a player in a game by a "substitute." This will help avoid the common mistake of keeping the letter instead of replacing it.

Insist on good systematic form of evaluating formulas.

Use as many algebraic applications as possible, such as:

What is the area of the part of the figure which is not shaded?



Make tables of corresponding values for certain formulas in order to stress the idea of dependence. One quantity of a formula depends on one or more others, as:

In the formula A = bh, A depends on both b and h. If either changes, A will also change.

Extend the work to other types of formulas, such as motion, cost and wages.

Have the pupils:

(1) Make formulas from verbal statements of rules.
(2) Derive a formula by beginning with specific examples with known values, gradually substituting literal values for each quantity, finally reaching a general statement. Test formula by many examples.

Goals	Materials and References
I. (Continued)	

I. (Continued)

(3) State the relationship of two or more quantities by means of a formula from a table of values gathered from many specific examples.

Solve problems involving the use of various formulas.

When a formula involves a geometric figure, have the pupils get the habit of sketching the figure.

Some teachers prefer to have formulas precede equations, others the opposite approach. This decision is left to the individual teacher.

The pupils should be taught to look upon the equation as a helpful device which enables them to state relationships which would be too complex to be investigated successfully or easily without its aid.

Equations are solved by certain fixed principles called "axioms." These should be explained arithmetically to the pupil and illustrated in terms of something familiar such as the balance scales.

Theseaxioms can be combined into the following single statement, called the Rule of Equality: "If equals are increased or decreased or multiplied or divided by equals, the results are equal". A sound procedure to follow in solving various types of equations is to develop a single unifying principle applicable to various forms. This eliminates a necessity for developing a special method for each form.

The principle of "elimination by undoing" or applying the opposite or inverse processes is not difficult if the pupils are shown its essential simplicity.

ELEMENTARY ALGEBRA 1
(First Semester)

	Goals	Materials and References
II. (2 wks)	A broader concept of our number system through the use of signed number	TEACHING OF SECONDARY MATHEMATICS by Butler and Wren. Pp. 293-300. HISTORY OF MATHEMATICS, Volume II, by D. E. Smith. Pp. 395-404. "The Laws of Signs in Multiplication" by James H. Zant. The Mathematics Teacher, April, 1939. Pp. 179-180. THE TEACHING OF SECONDARY MATHEMATICS by Hassler and Smith. Macmillan. 1937. Pp. 245-252.

I. (Continued)

Care should be taken that division by zero is excepted in any rule dealing with division. As soon as pupils are able to understand the steps involved, it is suggested that the "proof" that one equals two (the fallacy depending upon division by x-a where x = a) be given to them. They will enjoy it and it will help them to remember what errors may occur if divison by zero is permitted. One illustration to show why division by zero is not possible:

$$\frac{10}{10} = 1 \;;\quad \frac{10}{1} = 10 \;;\quad \frac{10}{.1} = 100 \;;\quad \frac{10}{.01} = 1000 \,.$$

Pupils can see that as the divisor approaches zero, the quotient becomes "large beyond all bounds."

Checking of equations is very important, and yet pupils often miss its significance. Two methods of checking are suggested:

To check the root $x = 3$ in the equation

$$5x + 1 = 16:$$

Does $5\cdot3 + 1 = 16$? Yes.

$$5\cdot3 + 1 \overset{?}{=} 16$$
$$16 = 16$$

II. Review the nature of signed numbers.

Have pupils suggest examples of signed numbers in everyday experiences such as: stock market report, football gains and losses, temperature, dates B. C. and A. D., longitude and latitude.

Although the number scale is probably the best device, it should not be the only one used to illustrate the opposite character of positive and negative numbers, and the arbitrary selection of zero. Some other devices which can be used are: thermometer, elevator, distances.

ELEMENTARY ALGEBRA 1
(First Semester)

Goals	Materials and References
III. (2 wks) The ability to perform the four fundamental operations with monomials and polynomials.	PROBLEMS IN TEACHING SECONDARY SCHOOL MATHEMATICS by Breslich. University of Chicago Press. 1938. Chapters 5, 8.

II. (Continued)

It should be made clear that whether a number is positive, negative or zero depends not only upon its own position on the scale but also upon the position of the number to which it is referred.

In the study of the fundamental processes with signed numbers, refer to the 8B course of study for content and teaching suggestions.

III. Develop the concept of monomial, binomial, trinomial, polynomial. Give examples of each.

An understanding of the following terms should grow out of a study of this unit: algebraic expression, factor, literal and numerical factor, coefficient, exponent, term, similar, dissimilar. Sufficient practice should be given to enable pupils to identify and use them understandingly. In teaching the addition and subtraction of monomials, include fractional and decimal coefficients.

Point out the method of combining dissimilar terms.

In developing the rule for multiplication of monomials, show that the order of a series of factors does not change the product, as:

$$(-3xy)\ (+2xy) = (-3)(+2)\ (x)(x)(y)(y) = -6x^2y^2$$

Make sure that pupils understand the use of exponents. Define exponent (positive integral) as the number of times the base is used as a factor.

Give practice with integral, fractional and decimal coefficients. In the division of monomials, check by multiplication.

Polynomials

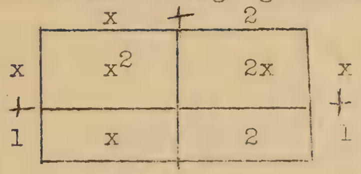
Work with polynomials has been greatly reduced in recent years. "Drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take.

Goals	Materials and References
III. (Continued)	

III. (Continued)

It must be conceived throughout as a means to an end and not as an end in itself." From Report of the National Committee on Mathematical Requirements. 1923. Pp. 11.

Multiplication and division of polynomials may be introduced through geometric illustration, as:



$x^2+2x+x+2=x^2+3x+2$

Compare arithmetic multiplication and division with multiplication and division of polynomials.

Additional practice in multiplication may be provided by using it in checking division.

Use some division examples that have remainders

Include examples in multiplication and division in which the terms must first be rearranged.

Show that parentheses, like other symbols of aggregation, are used to indicate a unity of the quantities enclosed. Show also that the removal or insertion of parentheses is possible only by some law of operation, as addition, subtraction or multiplication, and not merely by a change of signs. Avoid nests of parentheses.

ELEMENTARY ALGEBRA 1
(First Semester)

Goals	Materials and References
IV. (6 wks) A thorough knowledge and an appreciation of the equation as a powerful tool in problem solving acquired through:	TEACHING OF SECONDARY MATHEMATICS by Butler and Wren. Pp. 279-283, 305-307, 318-325.
A. Solving and checking equations of all types involving one unknown	
B. Solving many problems involving one unknown.	"The Function of Verbal Problems in Elementary Algebra," by Ellis. The Mathematics Teacher, May, 1943. "New Objectives for Ninth Grade Mathematics" by William Willits. The Mathematics Teacher, Dec., 1942. "Methods of Teaching Verbal Problems," J. A. Nyberg, Seventh Yearbook, National Council Teachers of Mathematics.

Teaching Suggestions

IV.

A. If short cuts such as changing sign, "cancelling," transposition or transformation are used in solving equations, insist that pupils understand the meanings of the processes involved. Transformation is a convenient way of applying the axioms of addition and subtraction. Use of the term "transposition" is misleading.

Give careful attention to special types of equations which often cause trouble because of unusual features, such as negative roots, parentheses, or fractional coefficients.

Insist upon the checking of all equations; substitute the root in the original equation. Where parentheses are involved remind pupils that the numbers within the parentheses must be combined first.

Teach pupils to solve and check literal equations and formulas by applying the same rules and axioms used in solving other equations. The check for literal equations except for those with very simple answers can be by numerical substitution.

B. Teachers should work towards a general method of analysis in problem solving rather than particular methods for special types of problems.

At first problems should be done in class with pupil and teacher working together. It is not advisable to give home assignments until the pupils have gained confidence in their ability to work problems.

Goals	Materials and References
IV. (Continued)	ALGEBRA-FIRST COURSE by Breslich. Laidlaw. 1939. Pp. 296-318, 334-351. PROBLEMS IN TEACHING SECONDARY SCHOOL MATHEMATICS by Breslich. University of Chicago Press. 1938. Chapter VIII. FIRST YEAR ALGEBRA by Schorling, Clark, Smith. Chapter X.
C. Solving and checking systems of equations involving two unknowns by:	
1. Graphs	Graph blackboard chart Graph paper
2. Addition or subtraction	
3. Substitution	

Goals	Materials and References
(Continued)	
D. Solving many problems involving two unknowns	"Use of Figures in Solving Problems in Algebra and Geometry" by Offa Neal. The Mathematics Teacher, May, 1940.

IV. (Continued)

With some problems, the tabular arrangement of the data is very effective. It is also a good plan to use diagrams. Geometric problems should be illustrated with the appropriate figure.

Although no specific methods can be taught for use in solving problems, the teacher should suggest a general procedure to be followed.

Make sure that pupils understand all words and concepts used in the problems.

Make sure that pupils check to see that they have answered the question asked in the problem, and that the answer or answers are correctly labeled.

Check the answer in the problem itself and not in the equation which was set up.

Teachers should be on the alert to find practica problems to supplement those found in the textbooks.

C. The graphic solution of equations having two unknowns arouses the interest of the pupils. It serves to explain the term "linear equation" and it clearly illustrates why the solution of a system of linear equations leads to one pair of numbers. Pupils, however, will need practice in plotting points and graphing single equations before graphing systems of equations. There are two disadvantages to the graphical method: (1) It is possible to get only approximate solutions instead of exact ones. This often causes a feeling of dissatisfaction as the sets of values wil not check in the equations. (2) It is a comparatively slow, tedious and inefficient method for solving linear equations. Therefore, pupils wil prefer the more exact and efficient algebraic methods when they are learned.

	Goals	Materials and References
. (3 wks)	A knowledge of special products and factors, including the ability to solve quadratic equations by factoring	"Factoring Has a Meaning," by H. Murtaugh. *The Mathematics Teacher*, December, 1941.

IV. (Continued)

An understanding of the following terms should grow out of a study of this unit: origin, abscissa, ordinate, coordinates, axes, linear equations, system of equations, inconsistent equations, dependent equations.

Emphasize neatness, and as great a degree of accuracy as possible with the tools available.

Stress the function concept by continually emphasizing that one of the variables depends on the other to which arbitrary values have been assigned.

Explain carefully and illustrate the construction of a graph. Make at least one at the blackboard and at the same time have pupils work along step by step on paper.

Since interpretation of the solution of the two equations is the main point of difficulty, call the attention of the pupils to the fact that if a point lies on two graphs at the same time, its coordinates must satisfy each equation. Consequently, the coordinates of this point are the solution of the system.

It is easier for the average pupil to learn the addition and subtraction method first. The substitution method will be used here only when the coefficient of one of the unknowns is unity.

D. Show the advantages of using equations with two unknowns in solving some problems. It is helpful to work out several problems using both methods, one unknown or two unknowns.

ELEMENTARY ALGEBRA 1
(First Semester)

Goals	Materials and References
V. (Continued)	
VI. (3 wks) The ability to perform the operations of multiplicatioh and division with algebraic fractions with increased understanding of the corresponding processes in arithmetic	THE TEACHING OF SECONDARY MATHEMATICS. Butler and Wren Pp. 308-315 "A Presentation of Fractions to Avoid Pitfalls in Arithmetic and Algebra," by L. E. Leete. _The Mathematics Teacher_, October, 1941.

V. As the applications of factoring are practically limited to work with fractions and the solution of quadratic equations, the work in special products and factoring should be restricted to :

(1) Squaring a binomial.
(2) Finding the product of the sum and the difference of two terms.
(3) Factoring a polynomial the terms of which contain a common monomial factor.
(4) Factoring trinomials of the form $x^2 + bx + c$.
(5) Factoring the difference of two squares.

The two fundamental requirements of factoring are:

(1) Knowledge of the type forms of the various special kinds of factors and their associated products.
(2) Ability to recognize, identify and associate particular cases and particular elements with the general types to which they belong.

Include numerical applications of special products and factoring such as, 38×42, etc.

Geometric and arithmetical illustrations help to rationalize the type form and will provide a basis by which the student can easily reconstruct them.

To be of much use, the type form must be learned thoroughly.

Take time to study the factors of numbers such as; 24, 20, 30, 36, 48. Illustration: $24 = 24 \times 1$, or 12×2, or 8×3, or 6×4.

Give a series of examples in which the last term of each trinomial is the same number, as:

$$\begin{array}{ll} x^2 + 14x + 24 & x^2 - 23x - 24 \\ x^2 + 25x + 24 & x^2 + 10x + 24 \\ x^2 - 11x + 24 & x^2 + 2x - 24 \end{array}$$

Goals	Materials and References
VI. (Continued)	

Teaching Suggestions

V. (Continued)

Sufficient drill and practice will help fix the forms in the mind of the pupil. It should not be mere mechanical repetition. Drill should be made meaningful, through constant illustration and applications to specific examples involving both literal and numerical terms.

Include non-factorable expressions.

To maintain skill in factoring, provide some opportunity for its use.

Use quadratic equations only as an example of the use of factoring. Limit the equations to expressions that can be readily factored.

The verbal problems should be no harder to analyze than others previously solved. The only added difficulty should be the equation which must be factored.

An understanding of the following terms should grow out of this unit: common factor, difference of two squares, factoring, trinomial, horizontal multiplication, special products, square of a binomial, trinomial square, equation of second degree, quadratic equation.

VI. As a point of departure, start with the pupil's knowledge of arithmetic fractions.

It is neither necessary nor desirable to include highly complicated fractional expressions. Confine t the work mainly to fractions with monomial denominators.

Remind pupils that a fraction is indicated division.

Help pupils understand that the methods used in dealing with algebraic fractions are generalizations ofthe methods used with arithmetic fractions. For instance, in reduction of fractions, emphasize the fact that the value of a fraction remains unchanged if its numerator and denominator are multiplied or divided by the same quantity.

VI. (Continued)

Frequent reference to this principle will prevent the occurrence of such errors as: $\frac{x - \cancel{4}}{\cancel{4}} = x$

Most of the errors made in reducing fractions can be avoided by requiring the pupil to write the numerator and denominator of the fraction in factored form, enclosing each factor in parentheses. Then divide numerator and denominator by the same quantity.

To raise fractions to higher terms, have pupils enclose the numerator and denominator of the original fraction in parentheses and then write the common multiplier in parentheses, as a multiplier of the numerator and multiplier of the denominator.

It is better to avoid the use of the words "cancel" and "cancellation" and speak of dividing the numerato and denominator by the same quantity.

Point out the analogy between multiplication of arithmetic and algebraic fractions.

In the multiplication of fractions, when the answer should be 1, pupils often write 0. Show that the answer cannot equal zero unless zero appears as a factor in the numerator.

Be sure that pupils understand that division is the same as multiplication by the reciprocal of the divisor. This principle should be clearly explained and numerous illustrations given.

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
I. (3 wks) The ability to perform the operations of addition and subtraction with algebraic fractions with increased understanding of the corresponding processes in arithmetic	THE TEACHING OF SECONDARY MATHEMATICS, Butler and Wren. Pp. 312-318.
II. (4 wks) use the equation as an algebraic tool acquired through: A. Solving fractional equations	

ELEMENTARY ALGEBRA 2
(Second Semester)

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
II. (Continued)	
B. Solving formulas for any one of the letters.	
C. Solving problems involving the use of fractional equations.	
III. An understanding of and the ability to use ratio, proportion and variation in solving problems	Newspaper and magazine articles containing references to ratios. AVIATION MATHEMATICS, Felker et al. Pp. 139-143.

Teaching Suggestions

I. The work in addition and subtraction of algebraic fractions will be a generalization of the principles used with arithmetic fractions.

Remind pupils that:
   (1) In order to add or subtract fractions, the denominators must be the same.
   (2) Multiplying or dividing numerator and denominator by the same quantity does not change the value of the fraction.

The pupils will need practice in:
   (1) Adding fractions with the same denominator.
   (2) Changing fractions to equivalent fractions with the desired denominator.
   (3) Using the horizontal form of adding and subtracting.

Remind pupils that complex fractions are merely an indicated division of two expressions.

II.

A. The pupil should soon come to understand that equations that contain fractions can be changed into equivalent equations not having fractions by multiplying both members by the same quantity.

If the expression "clearing an equation of fractions" is used, be sure that it is correctly interpreted by the pupils, and that they know the mathematical principles involved.

Insist on neatness and care in all written work, since many mistakes are due to carelessness.

Anticipate a need for space in which to insert the least common denominator by which the fractions are multiplied.

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
III. (Continued)	A SOURCE BOOK OF MATHEMATICAL APPLICATIONS, National Council of Teachers of Mathematics, Bureau of Publications, Teacher's College, Columbia University. Pp. 80-90 problems 20.01, 20.02, 20.04, 20.05, 20.06, 20.07, 20.08, 20.09, 20.13, 21.01, 21.03, 21.04.

Teaching Suggestions

II. (Continued)

Point out the differences between addition of fractions and the solution of fractional equations, as pupils often confuse the two:

(1) In finding the algebraic sum:
  - a. The form of the fraction may be altered but the value of each fraction remains the same.

  - b. The numerator and denominator may be multiplied by the same quantity.

(2) In solving the equation:
  - a. Both the form and value of a term may be changed, but the equality of the two members must be maintained.
  - b. When "clearing an equation of fractions," multiply all of the numerators by the same quantity.

B. Remind pupils that solving a formula for one of the letters is an extension of the work in literal equations. Insist on the same procedure as those used in solving equations.

For further suggestions, see Goal IV-A of 9A.

C. Refer to Goal IV-B of 9A for suggestions on this topic.

III. Review the concept of ratio as taught in 8B (see 8B course of study) emphasing the following facts:
  - (1) Ratios are used to compare quantities by division.
  - (2) The quantities must be expressed in the same unit.
  - (3) The same rules which govern fractions govern ratios.

III. (Continued)

Call pupils' attention to the fact that in everyday life changes constantly occur. Many of these changes can be expressed mathematically.

Develop this idea of dependence (or the function concept) by analyzing and discussing numerous examples of quantities which change in value as other quantities change. For example:

(1) The distance a car covers depends upon how fast it is going and how long it travels.
(2) The period of a pendulum depends on its length.

Review the concept of proportion as taught in 8B (see 8B course of study), recalling the following ideas:

(1) A proportion is a statement that two ratios are equal.
(2) In a proportion the product of the means is equal to the product of the extremes.

Develop the idea that similar figures are figures which have corresponding angles equal and corresponding sides proportional.

Give practice in solving problems using proportions, particularly those relating to similar triangles.

For superior pupils enrich the course by extending the ideas of proportion to direct, inverse, and joint variation.

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
IV. Familiarity with the nature and significance of numerical trigonometry as a branch of mathematics, acquired through: A. An appreciation of its uses as a means in indirect measurement	THE TEACHING OF JUNIOR HIGH SCHOOL MATHEMATICS by Smith and Reeve. Ginn & Co. 1927. Chapter VII. THE TEACHING OF SECONDARY MATHEMATICS by Hassler & Smith. Macmillan Co. 1937. Pp. 86-88. ALGEBRA THE LANGUAGE OF MATHEMATICS by J. R. Overman, Lyons and Carnahan. 1940. Pp. 123-124.
B. An understanding of the use of these natural trigonometric functions: (1) Tangent (2) Sine (3) Cosine	MATHEMATICS IN AVIATION by Osteyee. Pp. 131-138. FIRST YEAR ALGEBRA by Schorling et al. Chapter XVII. "Enrichment Material for First Year Algebra" by D. U. Star. The Mathematics Teacher, Feb. 1939. " A Device for Teaching the Concept of the Trigonometric Functions" by L. Blank. The Mathematics Teacher, February, 1939.

IV.

A. The study of numerical trigonometry affords an opportunity for pupils to increase their understanding of the function concept, and to use the algebraic processes already learned in new and significant problems.

Discuss the need for and use of indirect measurement. Illustrate through numerous problem situations how trigonometry can be used, as in engineering, surveying, astronomy, navigation, avigation.

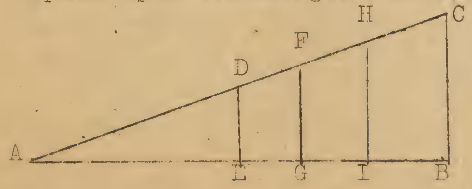
Point out the derivation of the word trigonometry (tri - three; gon - angle; metry - measure). Some brief comments on the history of trigonometry will do much to motivate the study.

B. Have pupils measure angles and sides of right triangles and compute the numerical value of the ratio representing the tangent. Graph paper is helpful here.

Stress the fact that all results obtained by measurement are approximate.

The rules for rounding off numbers are necessary at this point.

Illustrate that the function of an angle does not depend upon the lengths of its sides.



$$\frac{CB}{AB} = \frac{HI}{AI} = \frac{FG}{AG} = \frac{DE}{AE}$$

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
IV. (Continued)	
C. A knowledge of the use of trigonometric tables.	
D. The use of trigonometry in solving significant problems.	FIRST YEAR ALGEBRA, Schorling et al. Pp. 418. A SOURCE BOOK OF MATHEMATICAL APPLICATIONS, National Council of Teachers of Mathematics. Bureau of Publications. 1942. Pp. 218-244, problems 3.01, 3.02, 3.06, 3.07, 4.01, 4.02, 4.05, 4.06, 4.07, 4.08, 10.01, 10.02, 10.03, 12.03, 12.04, 12.05, 12.06, 12.07, 12.08, 12.09, 12.10, 12.12, 12.13, 12.15, 12.16, 12.17 FIELD WORK IN MATHEMATICS, Shuster and Bedford. American Book Co. 1935. Pp. 50-53, 77, 78, 82, 83, 104, 111, 118, 120. Instruments for Measuring Angles: THE TEACHING OF JUNIOR HIGH SCHOOL MATHEMATICS, Smith and Reeve. Pp. 360-362. FIELD WORK IN MATHEMATICS, Shuster and Bedford. Pp. 47-53, 39-46, 68-85, 98-132.

IV. (Continued)

Express the function in words as well as letters.

$$\text{Tangent } A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$$

C. After the tangent has been thoroughly understood and a reasonable degree of accuracy and skill developed, the sine and cosine functions should be studied in a similar manner.

Give practice in usihg trigonometric tables.

D. Develop the concept of an angle of elevation and angle of depression.

Give practice in using trigonometric functions in problem situations and suggest a definite procedure to be followed.

Use angle measurements in degrees only.

Problems used at the beginning should be so selected that the unknown quantity is in the numerator. Later, problems may be used with the unknown in the denominator.

All textbooks on the list furnish good problems. For additional ones see references. This topic furnishes an excellent opportunity for using problems from related fields such as: aviation, engineering, etc., particularly for superior groups.

The study of interpolation is optional.

Pupils can construct homemade transits and use them in actual field work. Such field work must be carefully planned and supervised.

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
V. An understanding of (2 wks) squares and square roots **through:**	
A. Ability to use tables of squares and square roots.	Homemade Instruments, JUNIOR MATHEMATICS FOR TODAY, Bk II by Betz. Ginn & Co. 1938. Pp. 190-192. AVIATION MATHEMATICS, Felker et al. Chapter VII.
B. Ability to find square root	
C. Its application to the Pythagorean theorem	AVIATION MATHEMATICS, Felker et al. Pp. 180-181.

V. Use this definition of square root: The square root of a number is one of its two equal factors.

A. Teach pupils to use square root tables to find squares and square roots and also to find:
    (1) Square roots of numbers in the square column. $\sqrt{12769} = 113$
    (2) Squares of decimals whose digits are the same as those in the number column. $(12.5)^2 = 156.25$
    (3) Square roots of decimals with an even number of decimal places, whose digits are the same as those in the square column. $\sqrt{201.64} = 14.2$
    (4) Squares and square roots of numbers having the same digits as those numbers in the table but with zeros attached.
    $(3800)^2 = 14440000$
    $\sqrt{5400} = 73.48$

B. Show that the table is often insufficient for certain needs and one must know how to find square roots arithmetically. Textbooks include an explanation of find a square root arithmetically.

Usually irrational roots should be expressed correct to hundredths. There are times when it is sufficient to have answers correct to tenths. Make the pupils realize that it is often necessary for them to decide what degree of accuracy is necessary.

C. Review the Pythagorean theorem. For suggestions, see 8B course of study.

Use problems of current interest. Teachers can find a wealth of problems that are real to the pupils, and in which they can see a practical value.

Have pupils use this method to measure indirectly some objects, either as a home assignment or as a class field project.

ELEMENTARY ALGEBRA 2
(Second Semester)

Goals	Materials and References
VI. (2 wks) A broadening concept of our number system through a knowledge of radicals based on: A. An understanding of the nature and significance of radicals B. The ability to perform the four fundamental operations with radicals C. The ability to solve simple radical equations	TEACHING OF SECONDARY MATHEMATICS, Butler and Wren. Pp. 329-336. MATHEMATICS FOR JUNIOR HIGH SCHOOL TEACHERS, Schaaf, Johnson Publishing Co. 1931. Pp. 409-413. MATHEMATICS IN AVIATION, Osteyee. Pp. 108-113.
VII. (3 wks) The ability to solve quadratic equations A. Types 1. $ax^2 = b$	FIFTEENTH YEARBOOK. Pp. 90 TEACHING OF SECONDARY MATHEMATICS. Butler and Wren. Pp. 343-345.

Teaching Suggestions

VI.

A. Emphasize that the square root of a number is one of its two equal factors, the cube root is one of the three equal factors, etc.

Teach pupils the precise meaning of rational and irrational numbers. Review with pupils the meaning and laws of positive integral exponents.

A knowledge of higher powers of some numbers is a timesaver:

$(2)^3 = 8$ $(2)^4 = 16$ $(2)^5 = 32$
$(3)^3 = 27$ $(3)^4 = 81$ $(4)^3 = 64$
$(5)^3 = 125$ $(5)^4 = 625$ etc.

B. If pupils are required to write out in detail all of the steps in the operations with radicals, many difficulties can be avoided.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16}\,\sqrt{3} = 4\sqrt{3}$$

C. It is not necessary to go beyond equations of the type $a\sqrt{bx - c} = d$ in difficulty.

VII.

A.

1. The FIFTEENTH YEARBOOK of the National Council of Teachers of Mathematics recommends that the required work with quadratic equations should be limited to the form $ax^2 = b$.

Goals	Materials and References
VII. (Continued)	
2. Quadratics solved by:	
* a. Graphs	
b. Factoring	
* c. Completing the square	
* d. The quadratic formula	
B. Solving significant problems involving quadratic equations	

VII. (Continued)

Introduce two new axioms to be used in solving equations of this type.

(1) Like roots of equal quantities are equal.
(2) Like powers of equal quantities are equal.

2.

a. The graphical method is not strictly an algebraic method, but is valuable in clarifying the meaning and nature of the two roots. It should not be required for many problems because it is slow and tedious.

b. The reason for setting the factors equal to zero should be made clear.

c. For the method of completing the square, review briefly the characteristics of the trinomial square, since it is the foundation of this method.

The principal function of completing the square is to provide a means for developing the quadratic formula.

d. The development of the quadratic formula should be carefully explained, and its advantages pointed out. Much practice should be provided in its use.

B. Significant problems involving quadratic equations of the type $ax^2 = b$ were encountered in applications of the Pythagorean theorem. Others should be provided.

NINTH GRADE

GENERAL MATHEMATICS 1
(First Semester)

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Goals	Materials and References
I. ARITHMETIC (7 wks) A. Accuracy and reasonable speed in the four fundamental processes with integers.	MATHEMATICS IN DAILY USE by Hart, Gregory and Schult. D. C. Heath. 1940. Chapters I and II.
	NEW APPLIED MATHEMATICS by Lasley and Mudd. Prentice Hall. 1937. Chapter I.

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Teaching Suggestions

I. ARITHMETIC

A. Diagnostic tests at the beginning of the course will inform both teacher and pupils just where remedial work is needed. The pupil must:

   a. See the need for improvement.
   b. Know the cause of his errors.
   c. Desire to correct them.

In the remedial work it is a good idea to have pupils work in groups or individually on their own weaknesses. It is discouraging to pupils to have to drill on work they know.

Teach various ways of checking in all the processes. Some suggested methods of checking: casting out nines, adding in the opposite direction, exchanging multiplier and multiplicand, the Civil Service check.

Give some practice in horizontal addition. This will be new to some pupils and must be presented accordingly.

Some short cuts can be fun and also will improve pupils' ability to compute. If pupils really understand the principles behind the short cuts they use, and if they form the habit of checking the answer by estimating, they will be sure that their answers are correct and they will form the habit of using such short cuts in all their computations. Suggested short cuts:

   (1) Multiply and divide by powers of 10.
   (2) Use of fractional equivalents.

In order to promote better achievement, stress accuracy, and insist upon legible figures and a systematic arrangement of work.

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Goals	Materials and References
I. ARITHMETIC (Continued)	
B. The ability to solve problems involving the four fundamental processes	Teacher's Manual to MATHEMATICS IN DAILY USE, Hart et al. Pp. 4-6. SOCIALIZED GENERAL MATHEMATICS by Hart and Gregory. D. C. Heath. 1937. Chapter IV.
C. A thorough understanding of the meaning of percentage based on:	
1. A knowledge of the meaning of per cent	
2. The ability: a. To find a per cent of a number b. To find what per cent one number is of another * c. To find a number when a per cent of it is given	Graph blackboard chart.
D. The ability to apply percentage to the solution of:	
1. Problems of everyday life	

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Goals	Materials and References
I. ARITHMETIC (Continued) 2. Business problems as: a. Discount b. Commission c. Profit and loss d. Interest	
II. MEASUREMENT AND (6 wks) INFORMAL GEOMETRY	MATHEMATICS IN LIFE by Schorling and Clark. World Book Co. 1937. Unit I.
A. An appreciation of the need for standard units of measurement based on the history of the development of such common units as foot, yard, and rod.	MATHEMATICS IN AVIATION by Osteyee. Macmillan. 1942. ALGEBRA: A WAY OF THINKING by Mitchell and Walker. Harcourt, Bruce and Co., 1936. Chapter X. American Council Pamphlets: THE STORY OF WEIGHTS AND MEASURES THE STORY OF OUR CALENDAR
B. An understanding of and the ability to use the following measures in practical problems: 1. Length 2. Area 3. Volume 4. Weight 5. Liquids 6. Time	HE MEASURES IN MILLIONTHS, Ford Motor Co. "The Master Art of Measurement" by Betz. Third Yearbook of National Council of Teachers of Mathematics. WHEN WE SPEAK OF A TENTH by Christopher Morley, and THE PURSUIT OF ACCURACY Hamilton Watch Co., Lancaster, Pennsylvania.

I. ARITHMETIC (Continued)

B. If good habits of attacking problems are developed pupils will not be frightened by new problems. The problems applying the four fundamental processes in arithmetic might well include those of real significance to the pupils, such as; School and home problems, problems of sewing and cooking, family purchases, etc. (The information which pupils get in many problems and the thinking and investigation stimulated by the problems presented, are as important, in many cases, as the computation).

Always have pupils estimate the reasonableness of the result.

C. Since slower children need much visual help, make use of a blackboard chart. If 100 squares are ruled off, many ideas in percentage can be made clearer, as for instance the difference between 50% and .5% which can be shown by shading corresponding parts of the whole.

Refer to the teaching suggestions concerning percentage in the 7th and 8th grade courses of study.

The following fractional equivalents should be made automatic: halves, thirds, fourths, fifths, eighths, tenths.

Some pupils might make scrapbooks to collect illustrations of uses of percentage. These might be source books for data for problems.

D. Keep the problems simple and within the pupils' experience. Practice in reading many problems and outlining procedure for solving them is very effective, i.e., analyzing procedure without the added problem of computation. One of the most common difficulties found among slower pupils is the lack of proper vocabulary. Be sure that pupils understand the words used in problems.

Teaching Suggestions

I. ARITHMETIC (Continued)

Consider discount from the standpoint of both merchant and purchaser.

In studying interest, various methods may be presented, but be sure that they master one.

II. MEASUREMENT AND INFORMAL GEOMETRY

A. Recall with pupils the early attempts at measurement with such units as cubit, digit, fathom, palm, span.

It might be well at this point for the teacher to paraphrase or read to the pupils "The Master Art of Measurement" from the Third Yearbook of the National Council, pages 151-154.

B. The Tables of Measures will be learned parrot-fashion unless the pupils estimate and pace off and measure lengths and learn through experience, for instance, how long a rod is, or a yard, or distances such as 25 feet (for estimating lengths along the street for parking purposes).

A strip of gummed paper marked off in inches and feet and pasted along the top of the blackboard will help pupils in estimating lengths.

Remind pupils that all measurement is approximate and that the amount of error depends on the instrument, the care used, and that which is measured.

Since pupils are all familiar with radios, the understanding of differences in time should be part of their education.

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Goals	Materials and References
III. GRAPHIC REPRESENTATION (3 wks)	MATHEMATICS IN DAILY USE by Hart et al. Chapter V.
A. An understanding of the value and importance of graphs through reading and interpreting graphs	MATHEMATICS FOR EVERYDAY AFFAIRS by Mallory. Sanborn. 1940. Chapter IV.
B. An ability to construct bar, line, circle, and picture graphs	

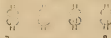
Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Teaching Suggestions

III. GRAPHIC REPRESENTATION

Graphs are an important part of our language of communication today, and everyone should be able to read them intelligently. Give much practice in interpreting graphs from everyday reading, since this ability is more important than the ability to construct graphs.

The study of graphs gives a fine opportunity for correlation of the mathematics work with other school subjects. Data for graphs are much more meaningful if they concern the pupils' lives or something they are studying in other school subjects. They will be able to supply additional data about themselves, their school, their city; they can bring some from their social studies work; or perhaps the mathematics class can illustrate the progress made in various school campaigns by graphs for bulletin boards or for the corridors.

By means of a number of small pictures or symbols (such as  etc.), a picture graph (also calle pictograph or pictogram) often shows a comparison more strikingly than broken lines or bars.

Give some practice in rounding off numbers. Do not assume that pupils understand it thoroughly, for they usually find it difficult. Sometimes it is helpful to start with rounding off measurements. Then go from small numbers to large ones.

Before starting this unit, make sure that each pupil is supplied with a few sheets of graph paper. The sheets can be cut into smaller pieces as needed

Ninth Grade
GENERAL MATHEMATICS 1
(First Semester)

Goals	Materials and References
IV. SCALE DRAWING (2 wks) The ability to read and make scale drawings	Blueprints Maps Charts MATHEMATICS IN LIFE by Schorling and Clark. Unit III. MATHEMATICS FOR AIR CREW TRAINEES, War Department Technical Manual 1-900. Section IV: Scales. MATHEMATICS IN DAILY LIFE by Barker and Morgan. Houghton Mifflin. 1939. Chapter II: Measuring and Drawing to Scale.
V. SUPPLEMENTARY PROJECTS The ability to apply various mathematical skills in a single project (No time allotment is given as projects will be carried out whenever feasible)	

Teaching Suggestions

IV. SCALE DRAWING

Scale drawings are so widely used that learning to read them is a necessary part of general education. Usually scale drawings are much smaller than the original, but often in nature it is desirable to make scale drawings larger than the objects. Call attention to the drawings and pictures made to scale in dictionaries.

Encourage pupils to bring examples of scale drawing to class. Those taking some shop courses will be able to bring good illustrations. Many can bring blue prints, charts, aviation maps.

Pupils might construct, to scale, the classroom, the school grounds, a football field, or tennis court.

Also, see references to scale drawing in the 8B course of study.

V. SUPPLEMENTARY PROJECTS

Ideas that have worked

Woodshop pupils constructed a small house to scale. This suggests other models which might be constructed to scale.

A group of girls studying home economics planned a luncheon, purchased the food, prepared and served it. They emphasized the skills and knowledge gained in their general mathematics class.

Boys taking printing exhibited their work to the mathematics class and explained how their knowledge of measurement and arithmetical computation was helpful in their work in printing.

NINTH GRADE

GENERAL MATHEMATICS 2
(Second Semester)

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Goals	Materials and References
I. ARITHMETIC (12 wks)	LEARNING TO COMPUTE by Schorling, Clark, Potter, Deady. World Book Co. 1940
A. Increased facility with the fundamental processes	
B. The ability to use arithmetic functionally in solving problems of:	
1. Leisure time concerning: a. Games b. Sports c. Puzzles d. Travel e. Amusements	MATHEMATICS IN EVERYDAY USE by Hart et al. Chapter VI.
2. Family finance a. Budgets b. Good buymanship c. Installment buying	CONSUMER MATHEMATICS by A. L. Cowan. Telegraph Press. Harrisburg, Pa. 1938. Public Affairs Pamphlets, as 45. HOW MONEY WORKS 50. CREDIT UNIONS Public Affairs Committee, 30 Rockefeller Plaza, N. Y.

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Teaching Suggestions

I. ARITHMETIC

A. Pupils like to see if they are making progress. Since the improvement of arithmetic skills is one of the important aims of this course, it would be a good idea to diagnose achievement in the various skills, provide for remedial work, and give maintenance tests.

B.

1. Much discussion is heard of the wise use of leisure time. Through the study of the mathematics of some leisure time activity, pupils may be helped toward a wiser use of such time and to a greater appreciation of a knowledge of mathematics.

a,b. The regulation dimensions of the playing field in football are 300 ft. by 160 ft. These figures may be useful in scale drawing.

d. The fare for traveling varies because of taxes and other reasons. It would be a good idea to have pupils investigate such rates at the time travel problems are studied.

e. It is hoped that problems concerning the mathematics of games of chance will make pupils more alert and make them see that the odds are against them in most of such games.

2. Thrift means wise spending as well as saving. This unit should point out many wise ways of spending and investing. The informational value of such work is very important as well as the computational values. Thus pupils should become more intelligent about family finances as well as more able to make the necessary computations.

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Goals	Materials and References
I. ARITHMETIC (Continued)	
d. Government securities	
e. Insurance	
f. Savings	
g. Owning and renting a home	
3. Community activities a. Protection - fire, police, etc.	EVERYDAY MATHEMATICS by Douglass and Kinney. Henry Holt. 1940. Chapter II: Government Activities and Expenditures.
b. Education and recreation	Public Affairs Pamphlet No. 28 OUR TAXES AND WHAT THEY BUY
c. Social welfare	
d. Taxes to meet these expenditures	

Teaching Suggestions

I. ARITHMETIC(Continued)

Pupils have helped in shopping for their parents and will be able to add practical ways of saving money in buying food and clothing.

Government regulations concerning installment buying have changed recently. Pupils will be able to bring to class the latest regulations because such information is furnished by merchants. The subject is included for the purpose of having pupils know about the practice, the extra charges necessarily incurred, and the importance of avoiding such buying when possible.

The study of insurance should include much material of social significance. The mathematics work will illustrate the general principles taught.

3. The informational content of this topic will correlate well with a course in citizenship. Through a study of community activities, pupils will learn about the advantages gained from them, and also their own responsibility in supporting such activities.

When pupils realize what people get for their tax money, they will see that paying taxes is not a burden but a privilege and a civic responsibility.

Have pupils find out the tax rate in this city, and also how it is determined. If the pupils are to be intelligent citizens, they must know how taxes are determined, collected, and spent. This work can be motivated by having the class represent a community and work out its financial problems.

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Goals	Materials and References
I. ARITHMETIC (Continued)	
4. Business transactions	
a. Buying and selling	
b. Banking practices	
(1) Services rendered by banks	
(2) Bank forms	
(3) Borrowing and lending money	
II. ALGEBRA (6 wks)	
A. An appreciation of algebra as the "shorthand of mathematics".	ALGEBRA AN INTERESTING LANGUAGE by E. Breslich. Newson, 1939. THE TEACHING OF SECONDARY MATHEMATICS by Butler and Wren. McGraw Hill. 1941.
B. An understanding of the language of algebra through:	MATHEMATICS IN AVIATION by Osteyee. Macmillan. 1942. See also references in Elementary Algebra Course of Study.
(1) Evaluating simple algebraic expressions.	
(2) Expressing statements by means of signs, letters, and numbers.	

Teaching Suggestions

I. ARITHMETIC (Continued)

4. This unit should help pupils become more intelligent consumers by learning various phases of business transactions.

   A working vocabulary of business terms should grow out of this study. It should include: Overhead, margin, invoice, profit, marked price, selling price; and these banking terms: Depositor, payee, face of check, endorse, canceled check, monthly statement, principal, interest, amount, promissory note, date of maturity, bank discount, net proceeds.

   Even though most of the pupils will not go into business themselves, this work will give them a better understanding of practices which they meet as consumers. Practically all of them will have business with banks and will need to be intelligent about them.

II. ALGEBRA

A. Since the language of algebra is another means of communicating quantitative ideas, this unit is included. It should give the pupils a glimpse into the field of algebra, so that if they decide to study it later, the basic concepts will be thoroughly understood.

B. For suggestions concerning the introduction of the symbolism of algebra, see the 8B and elementary algebra courses of study.

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Goals	Materials and References
II. ALGEBRA (Continued)	
C. An understanding of the formula as a mathematical tool acquired through:	
(1) Writing formulas to express rules	
(2) Evaluating simple formulas	
(3) Using formulas in solving practical problems	
D. The ability to solve simple equations	
E. A knowledge of the power of the equation as a tool in problem solving, gained through solving many significant problems	
F. An increased knowledge of the number system obtained through:	
(1) An appreciation of the need for negative numbers	

Teaching Suggestions

II. ALGEBRA (Continued)

C. Have pupils acquire the habit of evaluating formulas by four steps, namely:
   (1) Copying the formula
   (2) Writing down what is given
   (3) Making the substitutions (and performing the computation)
   (4) Writing the answer

They will have an intelligent way of evaluating all formulas, whether they be from geometry, physics, navigation, radio, or the like. Many pupils find difficulty in substituting correctly in the third step. For such persons, it is a good idea to have them actually copy the formula in the third step, cross out the letters, write the numerical value above each, and then do the computations indicated.

D. Emphasize the principle that equations remain equations if the same number is applied by the same process to both members of them. That is, teach principles rather than tricks of getting the answer.

E. In teaching the use of equations to solve problems, emphasize the fact that equations are merely a symbolic language for expressing the problem. Give practice in writing equations by reading problems to the class, word by word, and have pupils translate them immediately into equations.

F. New concepts in mathematics take time to learn. Give pupils time to develop a real understanding of the concept of signed numbers. Help them realize that negative numbers are merely another extension of the number system to meet certain needs and give meaning to solutions of certain equations.

Ninth Grade
GENERAL MATHEMATICS 2
(Second Semester)

Goals	Materials and References
II. ALGEBRA (Continued)	
(2) Familiarity with the uses of signed numbers	
(3) Ability to perform the four fundamental processes with signed numbers	
G. Familiarity with the meaning of squares and square root and the ability to use square root tables.	
H. Increased knowledge of indirect measurement through the use of the Pythagorean Theorem.	

II. ALGEBRA (Continued)

G. Refer to the 8B course of study.

H. Some historical notes on the Egyptian "rope stretchers" and their use of the 3-4-5 triangle would add to the interest in this topic.

Have pupils construct a 3-4-5 triangle with a square on each side and discover the Pythagorean relationship by actually counting the squar units. This procedure could be repeated with triangles whose sides are 5, 12, 13 or 8, 15, 17

Give practice in using the rule of Pythagoras in solving practical problems.

APPENDIX

TERMS AND SYMBOLS *

The Commission believes that it is unnecessary to furnish anything like an "Authorized List" of fundamental terms, since such a list might be in danger of interpretation as both prescriptive and restrictive. The sets of terms that appear in current textbooks are in large part neither too extensive nor too narrow for pedagogical purposes.

For the sake of unifying mathematics, and avoiding artificial distinction requiring the use of specialized terminology that may seem to the average pupil to be unnecessary or somewhat stilted, good psychological practice tends to carry over into geometrical discussion available algebraic notions rather than to preserve a separate system of terms derived from the Greeks whose knowledge of algebra was at best quite meager. For example, the special terms once common in discussing proportion, such as ANTECEDENT, CONSEQUENT, and symbols such as :: and the sign : for ratio seem inappropriate for present-day elementary instruction.

There are also trends of current style whose origin may not lie wholly in logic but which the teacher would do well to respect, and symbols used in other classes or outside of school contacts may sometimes be employed. Some special recommendations follow.

Geometry

UNDEFINED TERMS. The Commission recommends that no attempt be made to define formally terms of logic and geometry, such as ANGLE, CORRESPONDENCE, DIRECTION, DISTANCE, FIGURE, MAGNITUDE, NUMBER, PLANE, RELATION, SOLID, SPACE, STRAIGHT LINE, SURFACE, VARIABLE, although the significance of such terms should be made clear by informal explanation and discussions.

DEFINITE USAGE RECOMMENDED. It is the opinion of the Commission that the following general usage is desirable:

1. Line should mean COMPLETE STRAIGHT LINE. It has no finite length.

2. CIRCLE should be considered as the curve.

3. POLYGON (including TRIANGLE, SQUARE, PARALLELOGRAM, and the like) should be considered, by analogy to a circle, as a closed broken line. Similarly, SEGMENT OF A CIRCLE should be defined as the figure consisting of a chord and either of its arcs.

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* Extracts from Appendix III Fifteenth Yearbook, National Council of Teachers of Mathematics.

4. AREA OF A CIRCLE should be defined as the area (expressed in units of area) of the portion of the plane enclosed by the circle. AREA OF A POLYGON should be treated in the same way.

5. The usage recommended above with respect to plane figures is also recommended with respect to three-dimensional figures. For example, a SPHERE should be regarded as a surface, its VOLUME should be defined in a manner analogous to the area of a circle. A similar usage should be followed with respect to such terms as POLYHEDRON, CONE, and CYLINDER.

6. CIRCUMFERENCE should be considered not as a set of points but as the length (expressed in units of length) of the circle (line). Similarly, PERIMETER should be defined as the length of the broken line which forms a polygon; that is, as the sum of the lengths of the sides.

7. OBTUSE ANGLE should be defined as an angle greater than a right angle and less than a straight angle and therefore should not be defined merely as an angle greater than a right angle.

8. CONVEX (as opposed to CONCAVE), because of its wide application in science and the arts, may well be used and illustrated.

9. The term RIGHT TRIANGLE should be preferred to "right angled triangle," this usage being now so well standardized in this country that it may properly be continued in spite of the fact that it is not international. Similarly for ACUTE TRIANGLE, OBTUSE TRIANGLE, and OBLIQUE TRIANGLE.

10. Such English plurals as FORMULAS and POLYHEDRONS should be used in place of the Latin and Greek plurals.

11. It is not necessary or desirable to distinguish between AXIOM and POSTULATE. The word ASSUMPTION can be used in place of these words and may have more meaning to the pupil.

TERMS MADE GENERAL. It is the recommendation of the Commission that the modern tendency of having terms made as general as possible should be followed, although the special cases may continue to suggest separate discussion. For example:

1. ISOSCELES TRIANGLE should be defined as a triangle having two equal sides. There should be no limitation of two and only two equal sides.

2. RECTANGLE should be considered as including a square as a special case, although the notions of a square may be discussed first.

3. PARALLELOGRAM should be considered as including a rectangle, and hence a square, as a special case.

4. TRAPEZOID seems to be most frequently defined in textbooks as a quadrilateral with two and only two sides parallel; but in some cases the "only two" restriction is omitted (which causes a parallelogram to be a special case of a trapezoid). Consistently with the first definition, a TRAPEZIUM is defined as a quadrilateral of which no two sides are parallel. Some American dictionaries agree with such definitions, preserving the "only two" restriction in the trapezoid; but other dictionaries interchange the two definitions. The Commission recommends that TRAPEZOID be defined as first mentioned, and that the word TRAPEZIUM be dropped as unnecessary.

5. SEGMENT (in contrast to LINE) should be used to designate the part of a straight line included between two of its points (as well as the figure formed by an arc of a circle and its chord), this being the usage generally recognized by modern writers. (One obtains the length of a segment, never of a line, since a straight line extends indefinitely in each of two opposite directions.)

TERMS TO BE ABANDONED. It is the opinion of the Commission that the following terms are not of enough usefulness in elementary mathematics at the present time to make their recognition desirable in examinations, and that they serve chiefly to increase the technical vocabulary to the point of being burdensome and unnecessary.

1. ANTECEDENT AND CONSEQUENT.

2. EQUIVALENT. This is an unnecessary substitute for the more precise expressions "equal in area" and "equal in volume," or (where no confusion is likely to arise) for the single word "equal".

3. TRAPEZIUM

4. SCHOLIUM, OBLONG, SCALENE TRIANGLE, SECT PERIGON, RHOMBOID (the term OBLIQUE PARALLELOGRAM being sufficient), and REFLEX ANGLE (in elementary geometry).

5. SUBTEND (in certain connections). Probably pupils have been confused by the statement that an arc subtends a chord or an angle. In such connections the word HAS can be used. There seems, however, to be no adequate simple sutstitute for the word in such a statement as "A flagpole on top of a building subtends an angle of 10 degrees at a point, etc."

6. INTERCEPT. Since this term has special meaning in higher mathematics, it seems advisable to avoid it in elementary work.

Algebra

1. With respect to equations, the Commission calls attention to the fact that the classification according to degree is comparatively recent and that this probably accounts for the fact that the terminology is so unsettled. The Anglo-American custom of designating an equation of the first degree as a SIMPLE EQUATION has never been satisfactory, because the term has no real significance. The most nearly international terms are EQUATION OF THE FIRST DEGREE (or FIRST DEGREE EQUATION) and LINEAR EQUATION. The frequent use of both phrases will enlarge the pupil's understanding, though it is to be noted that the appropriateness of the name LINEAR EQUATION is not seen until the graph of such an equation is considered.

2. The term QUADRATIC EQUATION is well established. There should be, however, a clear understanding of the somewhat confusing etymology involved, and the use of the description SECOND DEGREE EQUATION should be neither proscribed nor banned. The terms PURE QUADRATIC, AFFECTED QUADRATIC, COMPLETE QUADRATIC, and INCOMPLETE QUADRATIC are fortunately disappearing.

3. As to other special terms, the Commission recommends that the use of the following be avoided in elementary instruction: EVOLUTION as a general description for finding a root; MULTIPLY AN EQUATION, CLEAR OF FRACTIONS, CANCEL AND TRANSPOSE, at least until the significance of the terms is entirely clear; ALIQUOT PART (except in commercial work).

4. The Commission also advises the use of either SYSTEM OF EQUATIONS or SET OF EQUATIONS rather than the phrase "simultaneous equations."

5. The term SIMPLIFY should not be used in cases where there is possibility of misunderstanding. For purposes of computation, for example, the form $\sqrt{8}$ may be simpler than the form $2\sqrt{2}$, and in some cases it may be better to express $\sqrt{\frac{3}{4}}$ as $\sqrt{0.75}$ instead of as $\frac{1}{2}\sqrt{3}$. In such cases, it is better to give more explicit instructions than to use the misleading term "simplify".

6. The Commission suggests that the word SURD need not be used for the expression IRRATIONAL NUMBER. It recognizes the difficulty generally met by young pupils in distinguishing between COEFFICIENT and EXPONENT, but it feels that it is undesirable to attempt to change these terms which have come to have a standardized meaning and which are reasonably simple. Considerations of a similar nature will probably lead to the retention of such terms as RATIONALIZE and EXTRANEOUS ROOT.

7. Since the word PLUS is associated with addition and MINUS with subtraction, the Commission feels that "positive" and "negative" numbers are to be preferred to "plus" and "minus" numbers. The fact that "plus" and "minus" serve also to characterize results of rounding off approximate numbers as in 17.2+, and 3.4- militates against the use of "plus number" for "positve number".

Arithmetic

1. While it is rarely wise to attempt to abandon suddenly the use of words that are well established in our language, the Commission feels called upon to express regret that very young pupils, often in the primary grades, are still required to use such terms as SUBTRAHEND, ADDEND, MINUEND, and MULTIPLICAND. Teachers seem rarely to understand the real significance of these words, nor do they recognize that they are comparatively modern additions to what used to be a much simpler vocabulary in arithmetic. The Commission recommends that such terms of theoretical arithmetic be used, if at all, only after the sixth grade.

2. Owing to the uncertainty attached to such expressions as "to three decimal places," "to thousandths," "correct to three places," "correct to the nearest thousandth," usage of the following kind is recommended: "To three decimal places" and "to thousandths" should be regarded as identical and as referring to a result carried only to thousandths without considering the figure of ten-thousandths. "CORRECT to three decimal places" and "CORRECT to the nearest thousandth" should be regarded as identical and as referring to a result which has been or might have been carried to ten-thousandths and then rounded off to show the nearest thousandth. It may be well to use "exact" in connection with the number of correct decimal places, using "precise" as described below.

3. Because of the appearance in many textbooks of careless definitions of "significant figures," the Commission suggests the following convention: When every figure in a number except the last is exactly correct and the error in the last is less than one-half unit, all the figures have actual meaning and are called significant figures. Zeros are significant when they come within this definition. When they are used simply to give place-value to other digits, zeros are not significant. It may be well to use "precise" in connection with the number of correct significant figures. For example, a measurement may be precise to one part in ten thousand.

4. The Commission recommends the use of the expression "numbers in standard form." By this is meant a numeral with the decimal point located after the first digit and this numeral multiplied, if necessary, by a power of ten to compensate for the change of position of the decimal point, such as $97000=9.7\times10^4$, $.00097=9.7\times10^{-4}$, $97^8\doteq7.8\times10^{15}$, $\frac{1}{987}\doteq1.013\times10^{-3}$.

SYMBOLS

Mathematics is characteristically symbolic. The importance of the general question of symbolism is well recognized--words are themselves symbols. For this section, however, we are interested in nonverbal symbols. There are involved questions of intelligibility and good taste. There can be no doubt that new symbols are being continually introduced. Many of them attain favor with the specialists or general mathematical public; more are eventually dismissed as personal idiosyncrasies. At the secondary school level, caution is usually to be preferred to unrestrained originality. One should hesitate to assume that new notation is called for extensively in the traditional branches of elementary mathematics. Symbolism that is misleading or merely unintelligible defeats its own purposes, and reasonable uniformity is demanded for maximum service. Consequently the Commission makes certain detailed suggestions regarding individual symbols and even goes so far as to propose a list from which teachers may select such as seem appropriate. Pupils may be encouraged to discuss their need for new symbols and devise as exercise material several symbols to be used in class. But they should ever keep in mind that symbols are not merely for the convenience of the writer. Unless the reader can be expected to be familiar with them, they fail to serve the first objective of communication. For habitual use in examinations or elsewhere, it is wise to be conservative and to use symbols familiar and acceptable to the public generally. From time to time mathematics teachers may be expected to extend approval to new symbols tried and accepted by experimental classes.

Pupils should not be allowed to form the habit of using unusual combinations of symbols and abbreviations. A teacher may well hesitate to approve such writing as the following: "2 boys bt. 2 doz. eggs @ 45¢/doz. and 3"@ 47¢/doz." However, on tests in which speed is being tested one may permit symbols and abbreviations as a sort of shorthand, provided that no ambiquity results.

Elementary mathematical symbols fall roughly into four types: (1) General literary or typographical symbols, such as . ? " : ; (2) those used throughout mathematics generally, such as the Arabic numerals, +, -, =, (), y, x, and the decimal point; (3) those used extensively in special parts of the theory but chiefly in informal classroom work; (4) standard commercial symbols that the student of mathematics encounters only in application to commercial problems, such as $, @, # (for pound or for number).

Geometry

1. The symbols $\triangle$ and $\bigcirc$ for triangle and circle are international, although used more extensively in the United States than in other countries. Their use, with appropriate plurals, is recommended.

2. The symbol $\perp$, representing the term PERPENDICULAR or the phrase IS PERPENDICULAR TO, is fairly international and the meaning is apparent. Its use is therefore recommended.

3. The symbol $\parallel$ for PARALLEL or IS PARALLEL TO is fairly international and is recommended.

4. The symbol $\sim$ for SIMILAR or IS SIMILAR TO is international and is recommended.

5. The symbols $\cong$ and $\equiv$ for CONGRUENT or IS CONGRUENT TO have considerable use in this country. The Commission feels that the former, which is fairly international, is to be preferred (in geometry) because it is the more distinctive and suggestive, particularly since $\equiv$ has been widely used for identity.

6. The symbol $\angle$ for ANGLE is coming to be generally preferred to any other because of its simplicity, and is therefore recommended.

7. Mathematical writers have not developed any symbols that have general acceptance for the following words, and there seems to be no necessity for making the attempt: SQUARE, RECTANGLE, PARALLELOGRAM, TRAPEZOID, QUADRILATERAL, SEMICIRCLE. (Certain obvious symbols suitable in print, would produce confusion if written carelessly.)

8. The symbol $\overset{\frown}{AB}$ for ARC AB cannot be called international. While the value of the symbol $\frown$ in place of the short word ARC is doubtful, the Commission sees no objection to its use except possible difficulty in typesetting for publication.

9. The symbol $\therefore$ for THEREFORE has a value that is widely recognized, but the symbol $\because$ for SINCE is used so seldom that it should be abandoned.

10. With respect to the lettering of figures, the Commission calls attention, for purposes of general information, to a convenient method, found in certain European and in some American textbooks, of lettering triangles thus: Capitals represent the vertices, corresponding small letters represent opposite sides, corresponding small Greek letters represent angles, and the prime letters represent the corresponding parts of a congruent or similar triangle. There is much merit in the plan, and the Commission is prepared to recommend it, with optional use of the Greek letters.

11. In general, it is recommended that a single letter be used to designate any geometric magnitude, whenever there is no danger of ambiguity. The use of numbers alone to designate non-numerical magnitudes should be avoided by the use of subscripts or accents, as in A_1, A_2, a', b''.

12. With respect to the symbolism for limits, the Commission calls attention to the fact that the symbol → (for "tends to") is both international and expressive and has constantly grown in favor in recent years. Although the subject of limits is not generaly treated scientifically in the secondary school, the idea is often mentioned in geometry and a symbol may occasionally be needed.

13. The Commission sees no objection to the use of the following abbreviations after the pupil has acquired facility in stating the theorems:
SSS for "three sides,"
ASS for "two sides and an angle adjacent to one of them,"
SAS for "two sides and the included angle," and
ASA for "two angles and the included side."

The words COPLANAR, COLLINEAR, and CONCURRENT have now acquired wide use and should be retained. Concerning the terms RAY and HALF-LINE the Commission wishes to be non-committal.

Algebra

The symbols in elementary algebra are now so well standardized as to require but few comments in a report of this kind. The Commission believes that it is desirable, however, to call attention to the following points:

1. Owing to the frequent use of the letter x, it is preferable in most cases to use the center dot (a raised period) for multiplication in the few cases in which any symbol is necessary. For example, in a case like $1 \cdot 2 \cdot 3 \cdots (x-1) \cdot x$, the center dot is preferable to the symbol × ; while in cases like $2a(x-a)$ no symbol is necessary. The Commission recognizes that the period (as in a.b) is more nearly international than the center dot (as in $a \cdot b$); but since the period will continue to be used in this country as a decimal point, it is likely to cause confusion, to elementary pupils at least, to attempt to use it also as a symbol for multiplication. As noted earlier one writes (in standard form)

   $97^8 = 7.8 \times 10^{15}$, to avoid possible confusion.

2. In recent years the "decimal" point has acquired new uses. For example, banks quote 95.17 for a Liberty Bond, meaning $95\frac{17}{32}$ and educational psychologists state a pupil's mental age as 12.3, meaning twelve years and three months. The Commission looks with disfavor upon such practices but admits their convenience. It is necessary to inform pupils of these practices.

3. With respect to division, the symbol '÷' is purely Anglo-American, the symbol : serving in most countries for division as well as ratio. Since neither symbol plays any large part in business life, it seems proper to consider only the needs of algebra, and to make more use of the fractional form and (where the meaning is clear) of the symbol /, and to drop the symbol ÷ in writing algebraic expressions. In the case of unusually long expressions for numerator and denominator, one may write, "A/B where A = ..., B=...., "or otherwise make clear the occurence of division.

4. With respect to the distinction between the use of + and - as symbols of operation and as symbols to distinguish positive and negative numbers, the Commission sees no reason for attempting to use smaller signs for the latter purpose, such an attempt never having received international recognition, and there is not sufficient need of two sets of symbols to warrant violating international usage and placing an additional burden on the pupil.

5. In connection with signs it is well to refer to the pedagogical question of explaining the rule of signs in products. One cannot explain, for instance, that a positive number multiplied by a negative number gives a negative number, by

saying that subtraction is involved. A correct explanation based upon the distributive law is simple, and throws much light upon the nature of algebra. The proof reveals that algebra resembles a game with the distributive law prescribed, and shows that important consequences can be derived from the law. Special cases can be used to advantage; for example, by applying the distributive law to the product $5(9 + (-3))$, which is known to be 30, it is readily found that $5(-3)$ must be -15. It is better not to attempt to prove the law of signs than to use fallacious reasoning that proves nothing and either confuses or misleads the pupil.

6. With respect to the distinction between the symbols $\equiv$ and $=$ as representing respectively identity and equality, the Commission calls attention to the fact that, while the distinction is generally recognized, the consistent use of the symbols is rarely seen in practice. The Commission recommends that the symbol $\equiv$ be not employed in examinations for the purpose of indicating identity. The teacher, however, should use both symbols if desired.

7. The Commission calls attention to and approves the use of the symbol $\approx$ for APPROXIMATELY EQUAL TO, the symbol having been introduced first by the engineering societies and having found wide use. For example: $\frac{639 \times 47}{32} \approx \frac{600 \times 50}{30} = 1000$

8. With respect to the root sign $\sqrt{}$, the Commission recognizes that convenience of writing assures its continued use in many cases instead of the fractional exponent. It is recommended, however, that in algebraic work involving complicated cases the fractional exponent be used. It should be emphasized that $\sqrt{a}$ (where a is a positive number) means only the positive root. To have ambiguity in such a symbol would be confusing, and good usage is clearly established. If the root sign itself indicated two numbers, it would not be necessary to use the sign of ambiguity, $\pm$, in the familiar formula for the quadratic equation, but that sign is always printed. Thus $\sqrt{4} = 2$, and it is not good usage to write $\sqrt{4} = \pm 2$. Similarly the two square roots of 3 are $\sqrt{3}$ and $-\sqrt{3}$. Analogous remarks can be made about $a^{\frac{1}{2}}$, $\sqrt[4]{a}$, $a^{\frac{1}{4}}$, etc.

When imaginaries are used, the symbol *i* should be employed instead of $\sqrt{-1}$ except possibly in the first presentation of the subject. The reason for writing $\sqrt{-a}$ (where *a* is positive) in the form $i\sqrt{a}$ becomes apparent upon considering multiplication. The product of $(i\sqrt{6})\,(i\sqrt{5}) = i^2\sqrt{6}\sqrt{5} = -\sqrt{30}$, which value might not be obtained if the numbers are left in the form $\sqrt{-6}$ and $\sqrt{-5}$, respectively. Finally it is to be noted that neither i nor $-i$ should be referred to as positive;"positive" implies "real."

9. As to the factorial symbols such as in 5! and $|\underline{5}$, to represent $5\cdot4\cdot3\cdot2\cdot1$, the tendency is very general to abandon the second one, probably on account of the difficulty of printing it. In such abandonment the Commission concurs. (This question is not, however, of great importance in the general courses in the high school.)

10. With respect to symbols for an unknown number there has been a noteworthy change within recent years. While the Cartesian use of *x* and *y* will doubtless continue for two general unknowns, the recognition that the formula is, in the broad use of ther terms, a central feature of algebra has led in the extended use of the initial letters of words to represent quantities. This is simply illustrated by the formula $A = \pi r^2$. The custom referred to is now international and should be fully recognized in the schools.

11. Unfortunately, it is still necessary to advise abandoning the double colon in proportion, and the symbol [illegible] in variation, although both of these symbols are practically obsolete in mathematics except in textbooks.

A MATHEMATICAL ATMOSPHERE IN THE CLASSROOM

"The appropriate furnishing and equipping of a mathematics classroom is receiving more attention than formerly, many teachers not being contented with four walls and a blackboard. The movement is partly a manifestation of the general wish to remove barrenness from schoolrooms and make them congenial places whose good cheer invites study and reflection. But it is also thought that proper equipment aids instruction, if indeed it is not essential for the type of teaching that has been made necessary by the present clientele of the schools." [1]

A bookshelf with interesting books and magazines is a fine addition to any classroom. Other ideas for helping to give a classroom a "mathematical atmosphere" are listed below.

BULLETIN BOARD AND WALL DISPLAYS

Tree of Knowledge
Pictures showing the history of measurement
Pictures of famous mathematicians (Some are available from Scripta Mathematica.)
Pictures or drawings of ancient mathematical instruments, as sun dials, ancient clocks, surveying and astronomical instruments
Mathematical clippings

Posters of the following sorts: [2]

1. Those which show the uses of mathematics in society.
2. Those which carry in their slogan a positive and definite suggestion for the pupil in connection with certain classroom tasks.
3. Those intended to stimulate younger pupils to greater endeavor.
4. Those showing geometric forms in nature and art.

Pictures of buildings and bridges
Blueprints
Graphs
Gunnery charts
Maps
Proverbs relating to mathematics

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[1] Fifteenth Yearbook of the National Council of Teachers of Mathematics. P. 238.

[2] See Third Yearbook of the National Council of Teachers of Mathematics. P. 269.

SUPPLIES

Colored chalk
Cross section paper
Graph blackboard chart
Chart, metric system (29-F-1 on D.C.School Supply List)
Scissors

MODELS

- Models of solid figures (student-made and others):
  - Prisms, pyramids, cylinders, cones, spheres, the 5 regular polyhedrons
- Sun dial
- Dissected cone
- Cone and cylinder of same height and base
- Prism and pyramid of same height and base

INSTRUMENTS

Desk protractors and compasses
Blackboard tools: protractor, rulers, triangles, T square
Rulers: English, decimalized English, metric
Student-made instruments, as transit, clinometer, etc.
Pantograph
Abacus
Napier's bones
Slide rule

MISCELLANEOUS

- Kaleidoscope
- Collections of:
  - Tiles in geometric shapes
  - Buttons in geometric shapes
  - Rock crystals in geometric shapes
- Geometry flash cards
- String designs based on geometric figures

SOME GOOD REFERENCES FOR MATHEMATICS EQUIPMENT

"Equipment of the Classroom." Appendix IV of the Fifteenth Yearbook of the National Council of Teachers of Mathematics.

"A Mathematical Atmosphere" by Olive A. Kee. Third Yearbook of the National Council of Teachers of Mathematics.

"The Mathematics Collection" by George **Wolff.** Eighth Yearbook of the National Council of Teachers of Mathematics.

"Arousing and Maintaining Interest in Mathematics" and "Mathematical Equipment. Its Use and Care." THE TECHNIQUE OF TEACHIN SECONDARY SCHOOL MATHEMATICS by E. Breslich. University of Chicago Press. 1937.

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"Fugitive Materials" by Frances Mullen. *The Mathematics Teacher*, May, 1938.

"Enrichment Materials for First Year Algebra" by Fred L. Bedford. *The School Executive*, April, 1936.

"Vitalizing Geometry with Visual Aids" by Drake and Johnson. *The Mathematics Teacher*, February, 1940.

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APPENDIX
(Continued)

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2. Curriculum Problems in Teaching Mathematics
3. Selected Topics in the Teaching of Mathematics
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5. Teaching of Geometry
6. Mathematics and Modern Life
7. Teaching of Algebra
8. Teaching of Mathematics in the Secondary School
9. Relational and Functional Thinking in Mathematics
10. Teaching of Arithmetic
11. The Place of Mathematics in Modern Education
12. Approximate Computation
13. The Nature of Proof
14. The Training of Mathematics Teachers
15. The Place of Mathematics in Secondary Education
16. Arithmetic in General Education
17. A Source Book of Mathematics Applications
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